Chapter 2: Measuring the Signatures of Reality

To investigate whether a geometry structures the Universe into being, we must begin by measuring its most stable empirical traits: the constants of Nature. If each constant corresponds to a symmetry-constrained transform of the underlying manifold, then they can be thought of as structural residues of the geometry that underlies physical law.

If every unique constant reflects an aspect of the manifold we inhabit, then to measure them is to glimpse the hidden geometry that persistently structures the world into being. These constants are the signatures of the hyperbolic stage we are trying to understand.

As Lord Kelvin once remarked, "When you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind."[34]

The measured features of our world—the constants of Nature—form the strongest foundation for any theory of what the world is and how it is constructed. Each one is attached to some action, scale, relation, or transformation in physics. Each one constrains the kinds of stories reality will allow us to tell.

Every constant of Nature is a clue: a measured feature, a residue of transforamtion, a signature from the architecture beneath. And each one has its own discovery story—an account of how it was pulled from the world by curiosity, ingenuity, and measurement.

Let us trace how a few of these were found—beginning with c, the speed of light.

c

It is easy to forget that for most of human history, light was assumed to move instantaneously. No delay. No travel. Just presence. That illusion was broken not by conjecture, but by observing something about the moons of Jupiter.

In 1676, Ole Rømer, while charting the eclipses of Io, noticed a strange variation. When Earth was approaching Jupiter, the eclipses arrived early. When Earth was receding, they arrived late. The simplest explanation was that light took time to travel—its speed was finite. That was the first chisel strike against an ancient belief. Rømer had revealed the delay, though not its precise pace.

Then came James Bradley, who noticed that stars appeared to shift position as Earth moved. The effect, known as stellar aberration, hinted at Earth's motion relative to light of finite speed. And with that, a second chisel strike.

By the mid-1800s, the race to measure light's velocity had become a contest of ingenuity. Hippolyte Fizeau fired light through rotating gears. Léon Foucault bounced beams between mirrors. Each measurement drew the veil back further, slowly revealing its number to higher precision.

But it was Einstein who gave the number meaning. In his theory of special relativity, c became the invariant—the speed of causality, the structural ceiling of motion, the fixed ruler of all frames. It was no longer a property of light—it was a property of spacetime itself.

Today, the meter is defined by fixing c to exactly $299792458\ \mathrm{m/s}$, and the second is defined by another constant of Nature—the hyperfine transition frequency of Cesium-133, $\Delta_{\nu_{Cs}}$. The *meter* and the *second* are built into the atomic grammar of the world, anchor points in the architecture of what is.

$$\hbar$$

If the speed of light defines the scaffolding of spacetime, then the reduced Planck constant reveals the granularity of action—the quantum tick of the Universe's internal logic.

At the dawn of the 20th century, Max Planck was wrestling with a puzzle: Why do hot objects radiate light in the way they do? The equations of the day could model the low-frequency end of the spectrum, but at the ultraviolet end, the theory exploded into infinity. It made no sense. The math failed where reality persisted.

So, Planck did something bold. He asked: what if energy itself comes in packets—discrete steps rather than a smooth slope? He introduced a constant—h—to define the size of these steps, tying energy directly to frequency:

$$E = hf$$

Where $E =$ energy, $h =$ the Planck constant, and $f =$ frequency.

It was a desperate move. A formal trick to make the math behave. But it worked. Too well. The blackbody spectrum snapped into alignment.

The ultraviolet catastrophe vanished. And the continuous picture of energy exchange collapsed.

That constant—h, and its reduced form $\hbar = h/2\pi$—didn't just solve a thermodynamic mystery. It unveiled something deeper: that Nature itself operates in quantized steps. Between one state and another, there is always a minimum unit of change—a graininess to the world that is not due to limitation, but structure.

Einstein used Planck's idea to explain the photoelectric effect, showing that light itself could be thought of as a stream of quanta—photons—with energies set by $\hbar$. Later Niels Bohr used it to explain why electrons occupy discrete orbits, and Schrödinger built it into the wavefunction that describes matter.

The entire edifice of quantum mechanics is suspended from this single idea.

$\hbar$ is a structural constant. It defines the quantum of action—the scale that sets the smallest possible unit of interaction and change across time.

To act is to transform—to move from one configuration to another. $\hbar$ tells us how much action is required for such a transformation to even exist. It is the price of change in the Universe. The toll paid at every turn of quantum evolution.

As measurement techniques advanced—from atomic clocks to quantum Hall experiments—our knowledge of $\hbar$ grew sharper. But its presence was always more than numerical. It was ontological. It said something about how the world must behave if it is to endure through time, structured by rule. $\hbar$ is a threshold. Below it, persistence becomes undefined. It is the clearest sign we have that persistence itself is quantized.

m_e

Where c sets the structural ceiling of causality, and $\hbar$ quantizes action, the electron mass—m_e—anchors the architecture of matter.

The electron is the first truly elemental particle we encountered—glimpsed in the flickering shadows of cathode ray tubes. In the late 19th century, J.J. Thomson was studying the glow inside vacuum tubes, curious about what made the phosphor walls flash. By bending the rays with electric and magnetic fields, he measured their charge-to-mass ratio. The result was astonishing: whatever these particles were, their mass was tiny, their charge significant, and their behavior followed precise, quantifiable rules.

He had discovered the electron—not by sight, but by inference. It revealed itself through geometric deflection.

But to know its mass, we needed another step. In the early 20th century, Robert Millikan performed his famous oil-drop experiment, suspending tiny droplets in an electric field. By adjusting the voltage until a drop hovered motionless, he could deduce the electron's charge. Plugging the value of that charge into Thomson's ratio gave the first glimpse of the electron's mass. That number—m_e—has become one of the most precisely known quantities in all of physics. It sets the characteristic scale of atomic structure, fixes the energy levels of electrons in atoms, governs scattering processes, and determines the size of matter itself.

This mass defines how tightly electrons cling to nuclei, how photons scatter off particles, how fields curve around charge. It is one of the constants that calibrates the abstract symmetry of the Dirac equation into a stable, material world.

In the Dirac equation, c, $\hbar$, and m_e calibrate the relativistic quantum dynamics of the electron: c fixes the spacetime scale, $\hbar$ fixes the quantum-action scale, and m_e fixes the rest-energy scale.

$$\left(i\hbar c\,\gamma^{\mu}\,\partial_{\mu} - m_e c^2\right)\psi = 0$$

Every constant of Nature anchors our theories, constrains their freedoms, and reveals a possible contour of the geometric structure behind atomic logic. The Committee on Data for Science and Technology (CODATA) maintains the most precise values of these parameters. Their list of constants, drawn from global collaboration and experiment, represents our sharpest empirical window into the quantitative structure of the Universe.

What follows is the complete list of the constants of Nature as curated by CODATA.[35] These are the numerical features of the world: empirical signatures, refined through generations of experiment, that together constitute the most faithful numerical portrait we possess of Nature's internal logic.

There are exactly 288 of these constants. That number might seem arbitrary now—but the chapters ahead will show why it is structurally significant—and how it is tied to the hyperbolic figure-eight knot.

Look closely and you will notice that the second, meter, coulomb, kelvin and kilogram appear again and again in the names of the constants. These are the atomic units through which physical law becomes measurable and coherent. Note: $\text{hertz} = 1/\text{second}$.

What follows is the working vocabulary of Nature's internal logic. Like any new language, it at first appears as noise. But with the right lens,

pattern emerges. Before we dive into their geometric origins, here are the constants themselves—each a keyhole into the structure of the cosmos.

the constants of Nature via CODATA

	name	symbol and value
1	alpha particle mass	$m_\alpha = 6.6446573450(21) \times 10^{-27}$ kg
2	alpha particle mass energy equivalent	$\mathrm{E}_\alpha = 5.9719201997(19) \times 10^{-10}$ J
3	alpha particle mass energy equivalent in MeV	$\dot{\mathrm{E}}_\alpha = 3.7273794118(12) \times 10^{3}$ MeV
4	alpha particle mass in u	$m_\alpha/\mathrm{A_{mass}} = 4.001506179129(62)$ u
5	alpha particle molar mass	$\mathrm{M}_\alpha = 4.0015061833(12) \times 10^{-3}$ kg/mol
6	alpha particle relative atomic mass	$\mathrm{A_r}(\alpha) = 4.001506179129(62)$
7	atomic mass constant atomic mass unit-kilogram relationship unified atomic mass unit	$\mathrm{A_{mass}} = 1.66053906892(52) \times 10^{-27}$ kg
8	atomic mass constant energy equivalent in MeV	$\dot{\mathrm{E}}_{\mathrm{A_{mass}}} = 9.3149410372(29) \times 10^{2}$ MeV
9	atomic mass unit-electron volt relationship	$\mathrm{A_{mass}} \vdots \mathrm{eV} = 9.3149410372(29) \times 10^{8}$ eV
10	atomic mass unit-hartree relationship	$\mathrm{A_{mass}} \vdots E_h = 3.4231776922(11) \times 10^{7}\ \mathrm{E_h}$
11	atomic mass unit-hertz relationship	$\mathrm{A_{mass}} \vdots \mathrm{Hz} = 2.25234272185(70) \times 10^{23}$ Hz
12	atomic mass unit-inverse meter relationship	$\mathrm{A_{mass}} \vdots 1/\mathrm{m} = 7.5130066209(23) \times 10^{14}$ 1/m
13	atomic mass unit-joule relationship atomic mass constant energy equivalent	$\mathrm{A_{mass}} \vdots \mathrm{J} = 1.49241808768(46) \times 10^{-10}$ J
14	atomic mass unit-kelvin relationship	$\mathrm{A_{mass}} \vdots \mathrm{K} = 1.08095402067(34) \times 10^{13}$ K
15	atomic unit of 1st hyperpolarizability	$\mathrm{A_{1^{st}\,hp}} = 3.2063612996(15) \times 10^{-53}\ \mathrm{m^3\ C^3/J^2}$
16	atomic unit of 2nd hyperpolarizability	$\mathrm{A_{2^{nd}\,hp}} = 6.2353799735(39) \times 10^{-65}\ \mathrm{m^4\ C^4/J^3}$
17	atomic unit of charge density	$\mathrm{A_{cd}} = 1.08120238677(51) \times 10^{12}\ \mathrm{C/m^3}$
18	atomic unit of current	$\mathrm{A_{current}} = 6.6236182375082(72) \times 10^{-3}$ A
19	atomic unit of electric dipole momentum	$\mathrm{A_{edm}} = 8.4783536198(13) \times 10^{-30}$ m C
20	atomic unit of electric field	$\mathrm{A_{ef}} = 5.14220675112(80) \times 10^{11}$ V/m
21	atomic unit of electric field gradient	$\mathrm{A_{ef\,\nabla}} = 9.7173624424(30) \times 10^{21}\ \mathrm{V/m^2}$
22	atomic unit of electric polarizability	$\mathrm{A_{ep}} = 1.64877727212(51) \times 10^{-41}\ \mathrm{m^2\ C^2/J}$
23	atomic unit of electric potential	$\mathrm{A_{e\,pot}} = 2.7211386245981(30) \times 10^{1}$ V
24	atomic unit of electric quadrupole momentum	$\mathrm{A_{eqm}} = 4.4865515185(14) \times 10^{-40}$ m C
25	atomic unit of force	$\mathrm{A_{force}} = 8.2387235038(13) \times 10^{-8}$ N
26	atomic unit of length Bohr radius	$a_0 = 5.29177210544(82) \times 10^{-11}$ m
27	atomic unit of magnetic dipole moment	$\mathrm{A_{mdm}} = 1.85480201315(58) \times 10^{-23}$ J/T
28	atomic unit of magnetic flux density	$\mathrm{A_{mfd}} = 2.35051757077(73) \times 10^{5}$ T
29	atomic unit of magnetizability	$\mathrm{A_{mag}} = 7.8910365794(49) \times 10^{-29}\ \mathrm{J/T^2}$
30	atomic unit of momentum	$\mathrm{A_{mom}} = 1.99285191545(31) \times 10^{-24}$ m kg/s
31	atomic unit of permittivity	$\mathrm{A_{perm}} = 1.11265005620(17) \times 10^{-10}$ F/m
32	atomic unit of time	$\mathrm{A_{time}} = 2.4188843265864(26) \times 10^{-17}$ s
33	atomic unit of velocity	$\mathrm{A_{vel}} = 2.18769126216(34) \times 10^{6}$ m/s
34	Avogadro constant	$N_A = 6.02214076 \times 10^{23}$ 1/mol
35	Bohr magneton	$\mu_B = 9.2740100657(29) \times 10^{-24}$ J/T
36	Bohr magneton in eV/T	$\mu_B/e = 5.7883817982(18) \times 10^{-5}$ eV/T
37	Bohr magneton in Hz/T	$\mu_B/h = 1.39962449171(44) \times 10^{10}$ Hz/T
38	Bohr magneton in inverse meter per tesla	$\mu_B/hc = 4.6686447719(15) \times 10^{1}$ 1/m T
39	Bohr magneton in K/T	$\mu_B/k_B = 6.7171381472(21) \times 10^{-1}$ K/T

40	Boltzmann constant	$k_B = 1.380649 \times 10^{-23}$ J/K
41	Boltzmann constant in eV/K	$\dot{k}_B = 8.617333262 \times 10^{-5}$ eV/K
42	Boltzmann constant in Hz/K	$\dot{k}_B = 2.083661912 \times 10^{10}$ Hz/K
43	Boltzmann constant in inverse meter per kelvin	$\dot{k}_B = 6.950348004 \times 10^{1}$ 1/m K
44	characteristic impedance of vacuum	$Z_0 = 3.76730313412(59) \times 10^{2}\ \Omega$
45	classical electron radius	$r_e = 2.8179403205(13) \times 10^{-15}$ m
46	Compton wavelength	$\lambda_C = 2.42631023538(76) \times 10^{-12}$ m
47	Coulomb's constant	$\kappa = 8.9875517923(14) \times 10^{9}$ m/F
48	conductance quantum	$G_0 = 7.748091729 \times 10^{-5}$ S
49	conventional value of ampere-90	$\mathrm{A_{90}} = 1.00000008887$ A
50	conventional value of coulomb-90	$\mathrm{C_{90}} = 1.00000008887$ C
51	conventional value of farad-90	$\mathrm{F_{90}} = 0.99999998220$ F
52	conventional value of henry-90	$\mathrm{H_{90}} = 1.00000001779$ H
53	conventional value of ohm-90	$\Omega_{90} = 1.00000001779\ \Omega$
54	conventional value of volt-90	$\mathrm{V_{90}} = 1.00000010666$ V
55	conventional value of watt-90	$\mathrm{W_{90}} = 1.00000019553$ W
56	deuteron-electron magnetic moment ratio	$\mu_{\mathrm{de}}/\mu_e = -4.664345550(12) \times 10^{-4}$
57	deuteron g factor	$g_{\mathrm{de}} = 8.574382335(22) \times 10^{-1}$
58	deuteron magnetic moment	$\mu_{\mathrm{de}} = 4.330735087(11) \times 10^{-27}$ J/T
59	deuteron mag mom to Bohr magneton ratio	$\mu_{\mathrm{de}}/\mu_B = 4.669754568(12) \times 10^{-4}$
60	deuteron mag mom to nuclear magneton ratio	$\mu_{\mathrm{de}}/\mu_N = 8.574382335(22) \times 10^{-1}$
61	deuteron mass	$m_{\mathrm{de}} = 3.3435837768(10) \times 10^{-27}$ kg
62	deuteron mass energy equivalent	$\mathrm{E_{de}} = 3.00506323491(94) \times 10^{-10}$ J
63	deuteron mass energy equivalent in MeV	$\dot{\mathrm{E}}_{\mathrm{de}} = 1.87561294500(58) \times 10^{3}$ MeV
64	deuteron mass in u	$m_{\mathrm{de}}/\mathrm{A_{mass}} = 2.013553212544(15)$ u
65	deuteron molar mass	$\mathrm{M_{de}} = 2.01355321466(63) \times 10^{-3}$ kg/mol
66	deuteron-neutron magnetic moment ratio	$\mu_{\mathrm{de}}/\mu_n = -4.4820652(11) \times 10^{-1}$
67	deuteron-proton magnetic moment ratio	$\mu_{\mathrm{de}}/\mu_+ = 3.0701220930(79) \times 10^{-1}$
68	deuteron relative atomic mass	$\mathrm{A_r(de)} = 2.013553212544(15)$
69	electron charge to mass quotient	$-\mathrm{e}/m_e = -1.75882000838(55) \times 10^{11}$ C/kg
70	electron-deuteron magnetic moment ratio	$\mu_e/\mu_{\mathrm{de}} = -2.1439234921(56) \times 10^{3}$
71	electron g factor	$g_e = -2.00231930436092(36)$
72	electron gyromagnetic ratio	$\gamma_e = 1.76085962784(55) \times 10^{11}$ 1/s T
73	electron gyromagnetic ratio in MHz/T	$\dot{\gamma}_e = 2.80249513861(87) \times 10^{4}$ MHz/T
74	electron magnetic moment	$\mu_e = -9.2847646917(29) \times 10^{-24}$ J/T
75	electron magnetic moment anomaly	$a_e = 1.15965218046(18) \times 10^{-3}$
76	electron mag mom to Bohr magneton ratio	$\mu_e/\mu_B = -1.00115965218046(18)$
77	electron mag mom to nuclear magneton ratio	$\mu_e/\mu_N = -1.83828197187 7(32) \times 10^{3}$
78	electron mass atomic unit of mass natural unit of mass	$m_e = 9.1093837139(28) \times 10^{-31}$ kg
79	electron mass energy equivalent natural unit of energy	$\mathrm{E_e} = 8.1871057880(26) \times 10^{-14}$ J
80	electron mass in u	$m_e/\mathrm{A_{mass}} = 5.485799090441(97) \times 10^{-4}$ u
81	electron molar mass	$\mathrm{M}_e = 5.4857990962(17) \times 10^{-7}$ kg/mol
82	electron-muon magnetic moment ratio	$\mu_e/\mu_\mu = 2.067669881(46) \times 10^{2}$
83	electron-neutron magnetic moment ratio	$\mu_e/\mu_n = 9.6092048(23) \times 10^{2}$
84	electron-proton magnetic moment ratio	$\mu_e/\mu_+ = -6.5821068789(19) \times 10^{2}$
85	electron relative atomic mass	$\mathrm{A_r(e)} = 5.485799090441(97) \times 10^{-4}$
86	electron to shielded helion mag mom ratio	$\mu_e/\mu_{\mathrm{he}}' = 8.6405823986(70) \times 10^{2}$
87	electron to shielded proton mag mom ratio	$\mu_e/\mu_+' = -6.582275856(27) \times 10^{2}$
88	electron volt electron volt-joule relationship	$\mathrm{eV} = 1.602176634 \times 10^{-19}$ J

89	electron volt-atomic mass unit relationship	$\mathrm{eV} : \mathrm{A_{mass}} = 1.07354410083(33) \times 10^{-9}$ u
90	electron volt-hartree relationship	$\mathrm{eV} : E_h = 3.6749322175665(40) \times 10^{-2}\ E_h$
91	electron volt-hertz relationship	$\mathrm{eV} : \mathrm{Hz} = 2.417989242 \times 10^{14}$ Hz
92	electron volt-inverse meter relationship	$\mathrm{eV} : 1/\mathrm{m} = 8.065543937 \times 10^{5}$ 1/m
93	electron volt-kelvin relationship	$\mathrm{eV} : \mathrm{K} = 1.160451812 \times 10^{4}$ K
94	electron volt-kilogram relationship	$\mathrm{eV} : \mathrm{kg} = 1.782661921 \times 10^{-36}$ kg
95	elementary charge over h-bar	$e/\hbar = 1.519267447 \times 10^{15}$ A/J
96	Faraday constant	$F = 9.648533212 \times 10^{4}$ C/mol
97	Fermi coupling constant	$G_\mathrm{F} = 1.1663787(06) \times 10^{-5}$ GeV^{-2}
98	first radiation constant	$c_1 = 3.741771852 \times 10^{-16}$ W m^2
99	first radiation constant for spectral radiance	$c_{1L} = 1.191042972 \times 10^{-16}$ W m^2
100	hartree-atomic mass unit relationship	$E_h : \mathrm{A_{mass}} = 2.92126231797(91) \times 10^{-8}$ u
101	hartree-electron volt relationship Hartree energy in eV	$E_h : \mathrm{eV} = 2.7211386245981(30) \times 10^{1}$ eV
102	Hartree energy atomic unit of energy hartree-joule relationship	$E_h = 4.3597447222060(48) \times 10^{-18}$ J
103	hartree-hertz relationship	$E_h : \mathrm{Hz} = 6.5796839204999(72) \times 10^{15}$ Hz
104	hartree-inverse meter relationship	$E_h : 1/\mathrm{m} = 2.1947463136314(24) \times 10^{7}$ 1/m
105	hartree-kelvin relationship	$E_h : \mathrm{K} = 3.1577502480398(34) \times 10^{5}$ K
106	hartree-kilogram relationship	$E_h : \mathrm{kg} = 4.8508702095419(53) \times 10^{-35}$ kg
107	helion g factor	$g_\mathrm{he} = -4.2552506995(34)$
108	helion magnetic moment	$\mu_\mathrm{he} = -1.07461755198(93) \times 10^{-26}$ J/T
109	helion mag mom to Bohr magneton ratio	$\mu_h/\mu_B = -1.15874098083(94) \times 10^{-3}$
110	helion mag mom to nuclear magneton ratio	$\mu_h/\mu_N = -2.1276253498(17)$
111	helion mass	$m_\mathrm{he} = 5.0064127862(16) \times 10^{-27}$ kg
112	helion mass energy equivalent	$\mathrm{E_{he}} = 4.4995394185(14) \times 10^{-10}$ J
113	helion mass energy equivalent in MeV	$\mathrm{E_{he}} = 2.80839161112(88) \times 10^{3}$ MeV
114	helion mass in u	$m_\mathrm{he}/\mathrm{A_{mass}} = 3.014932246932(74)$ u
115	helion molar mass	$\mathrm{M_{he}} = 3.01493225010(94) \times 10^{-3}$ kg/mol
116	helion relative atomic mass	$\mathrm{A_r(he)} = 3.014932246932(74)$
117	helion shielding shift	$\sigma_\mathrm{he} = 5.9967029(23) \times 10^{-5}$
118	hertz-atomic mass unit relationship	$\mathrm{Hz} : \mathrm{A_{mass}} = 4.4398216590(14) \times 10^{-24}$ u
119	hertz-electron volt relationship	$\mathrm{Hz} : \mathrm{eV} = 4.135667696 \times 10^{-15}$ eV
120	hertz-hartree relationship	$\mathrm{Hz} : E_h = 1.5198298460574(17) \times 10^{-16}\ E_h$
121	hertz-inverse meter relationship	$\mathrm{Hz} : 1/\mathrm{m} = 3.335640951 \times 10^{-9}$ 1/m
122	hertz-joule relationship	$\mathrm{Hz} : \mathrm{J} = 6.62607015 \times 10^{-34}$ J
123	hertz-kelvin relationship	$\mathrm{Hz} : \mathrm{K} = 4.799243073 \times 10^{-11}$ K
124	hertz-kilogram relationship	$\mathrm{Hz} : \mathrm{kg} = 7.372497323 \times 10^{-51}$ kg
125	hyperfine transition frequency of Cs-133	$\Delta_{\nu_\mathrm{Cs}} = 9.192631770 \times 10^{9}$ Hz
126	inverse meter-atomic mass unit relationship	$1/\mathrm{m} : \mathrm{A_{mass}} = 1.33102504824(41) \times 10^{-15}$ u
127	inverse meter-electron volt relationship	$1/\mathrm{m} : \mathrm{eV} = 1.239841984 \times 10^{-6}$ eV
128	inverse meter-hartree relationship	$1/\mathrm{m} : E_h = 4.5563352529132(50) \times 10^{-8}\ E_h$
129	inverse meter-hertz relationship	$1/\mathrm{m} : \mathrm{Hz} = 2.99792458 \times 10^{8}$ Hz
130	inverse meter-joule relationship	$1/\mathrm{m} : \mathrm{J} = 1.986445857 \times 10^{-25}$ J
131	inverse meter-kelvin relationship	$1/\mathrm{m} : \mathrm{K} = 1.438776877 \times 10^{-2}$ K
132	inverse meter-kilogram relationship	$1/\mathrm{m} : \mathrm{kg} = 2.210219094 \times 10^{-42}$ kg
133	inverse of conductance quantum	${G_0}^{-1} = 1.290640372 \times 10^{4}\ \Omega$
134	Josephson constant conventional value of Josephson constant	$K_J = 4.835978484 \times 10^{14}$ Hz/V
135	joule-atomic mass unit relationship	$\mathrm{J} : \mathrm{A_{mass}} = 6.7005352471(21) \times 10^{9}$ u
136	joule-electron volt relationship	$\mathrm{J} : \mathrm{eV} = 6.241509074 \times 10^{18}$ eV
137	joule-hartree relationship	$\mathrm{J} : E_h = 2.2937122783969(25) \times 10^{17}\ E_h$

138	joule-hertz relationship	$\mathrm{J} : \mathrm{Hz} = 1.509190179 \times 10^{33}$ Hz
139	joule-inverse meter relationship	$\mathrm{J} : 1/\mathrm{m} = 5.034116567 \times 10^{24}$ 1/m
140	joule-kelvin relationship	$\mathrm{J} : \mathrm{K} = 7.242970516 \times 10^{22}$ K
141	joule-kilogram relationship	$\mathrm{J} : \mathrm{kg} = 1.112650056 \times 10^{-17}$ kg
142	kelvin-atomic mass unit relationship	$\mathrm{K} : \mathrm{A_{mass}} = 9.2510872884(29) \times 10^{-14}$ u
143	kelvin-electron volt relationship	$\mathrm{K} : \mathrm{eV} = 8.617333262 \times 10^{-5}$ eV
144	kelvin-hartree relationship	$\mathrm{K} : E_h = 3.1668115634564(35) \times 10^{-6}\ E_h$
145	kelvin-hertz relationship	$\mathrm{K} : \mathrm{Hz} = 2.083661912 \times 10^{10}$ Hz
146	kelvin-inverse meter relationship	$\mathrm{K} : 1/\mathrm{m} = 6.950348004 \times 10^{1}$ 1/m
147	kelvin-joule relationship	$\mathrm{K} : \mathrm{J} = 1.380649 \times 10^{-23}$ J
148	kelvin-kilogram relationship	$\mathrm{K} : \mathrm{kg} = 1.536179187 \times 10^{-40}$ kg
149	kilogram-atomic mass unit relationship	$\mathrm{kg} : \mathrm{A_{mass}} = 6.0221407537(19) \times 10^{26}$ u
150	kilogram-electron volt relationship	$\mathrm{kg} : \mathrm{eV} = 5.609588603 \times 10^{35}$ eV
151	kilogram-hartree relationship	$\mathrm{kg} : E_h = 2.0614857887415(22) \times 10^{34}\ E_h$
152	kilogram-hertz relationship	$\mathrm{kg} : \mathrm{Hz} = 1.356392489 \times 10^{50}$ Hz
153	kilogram-inverse meter relationship	$\mathrm{kg} : 1/\mathrm{m} = 4.524438335 \times 10^{41}$ 1/m
154	kilogram-joule relationship	$\mathrm{kg} : \mathrm{J} = 8.987551787 \times 10^{16}$ J
155	kilogram-kelvin relationship	$\mathrm{kg} : \mathrm{K} = 6.509657260 \times 10^{39}$ K
156	lattice parameter of silicon	$a = 5.431020511(89) \times 10^{-10}$ m
157	lattice spacing of ideal Si (220)	$d_{220} = 1.920155716(32) \times 10^{-10}$ m
158	Loschmidt constant (273.15 K, 100 kPa)	$n_0 = 2.651645804 \times 10^{25}$ 1/m^3
159	Loschmidt constant (273.15 K, 101.325 kPa)	$n_1 = 2.686780111 \times 10^{25}$ 1/m^3
160	luminous efficacy	$K_{cd} = 6.83 \times 10^{2}$ lm/W
161	magnetic flux quantum	$\Phi_0 = 2.067833848 \times 10^{-15}$ Wb
162	molar gas constant	$R = 8.314462618$ J/mol K
163	molar mass constant	$\mathrm{M_{A_{mass}}} = 1.00000000105(31) \times 10^{-3}$ kg/mol
164	molar mass of carbon-12	$\mathrm{M_{12_C}} = 1.20000000126(36) \times 10^{-2}$ kg/mol
165	molar Planck constant	$\mathrm{M}_p = 3.990312712 \times 10^{-10}$ J/s mol
166	molar vol of ideal gas (273.15 K, 100 kPa)	$V_{m_0} = 2.271095464 \times 10^{-2}$ m^3/mol
167	molar vol of ideal gas (273.15 K, 101.325 kPa)	$V_{m_1} = 2.241396954 \times 10^{-2}$ m^3/mol
168	molar volume of silicon	$V_m(Si) = 1.205883199(60) \times 10^{-5}$ m^3/mol
169	muon Compton wavelength	$\lambda_\mu = 1.173444110(26) \times 10^{-14}$ m
170	muon g factor	$g_\mu = -2.00233184123(82)$
171	muon magnetic moment	$\mu_\mu = -4.49044830(10) \times 10^{-26}$ J/T
172	muon magnetic moment anomaly	$a_\mu = 1.16592062(41) \times 10^{-3}$
173	muon mag mom to Bohr magneton ratio	$\mu_\mu/\mu_B = -4.84197048(11) \times 10^{-3}$
174	muon mag mom to nuclear magneton ratio	$\mu_\mu/\mu_N = -8.89059704(20)$
175	muon mass	$m_\mu = 1.883531627(42) \times 10^{-28}$ kg
176	muon mass energy equivalent	$\mathrm{E}_\mu = 1.692833804(38) \times 10^{-11}$ J
177	muon mass energy equivalent in MeV	$\dot{\mathrm{E}}_\mu = 1.056583755(23) \times 10^{2}$ MeV
178	muon mass in u	$m_\mu/\mathrm{A_{mass}} = 1.134289257(25) \times 10^{-1}$ u
179	muon molar mass	$\mathrm{M}_\mu = 1.134289258(25) \times 10^{-4}$ kg/mol
180	muon-proton magnetic moment ratio	$\mu_\mu/\mu_+ = -3.183345146(71)$
181	natural unit of action in eV s	$\dot{\hbar} = 6.582119569 \times 10^{-16}$ eV s
182	natural unit of energy in MeV electron mass energy equivalent in MeV	$\dot{\mathrm{E}}_e = 5.1099895069(16) \times 10^{-1}$ MeV
183	natural unit of momentum	$\mathcal{N}_{\mathrm{mom}} = 2.73092453446(85) \times 10^{-22}$ m kg/s
184	natural unit of momentum in MeV/c	$\dot{\mathcal{N}}_{\mathrm{mom}} = 5.1099895069(16) \times 10^{-1}$ MeV/c
185	natural unit of time	$\mathcal{N}_{\mathrm{time}} = 1.28808866644(40) \times 10^{-21}$ s
186	neutron Compton wavelength	$\lambda_n = 1.31959090382(67) \times 10^{-15}$ m
187	neutron-electron magnetic moment ratio	$\mu_n/\mu_e = 1.04066884(24) \times 10^{-3}$

188	neutron g factor	$g_n = -3.82608552(90)$
189	neutron gyromagnetic ratio	$\gamma_n = 1.83247174(43) \times 10^{8}$ 1/s T
190	neutron gyromagnetic ratio in MHz/T	$\gamma_n = 2.91646935(69) \times 10^{1}$ MHz/T
191	neutron magnetic moment	$\mu_n = -9.6623653(23) \times 10^{-27}$ J/T
192	neutron mag mom to Bohr magneton ratio	$\mu_n/\mu_B = -1.04187565(25) \times 10^{-3}$
193	neutron mag mom to nuclear magneton ratio	$\mu_n/\mu_N = -1.91304276(45)$
194	neutron mass	$m_n = 1.67492750056(85) \times 10^{-27}$ kg
195	neutron mass energy equivalent	$\mathrm{E}_n = 1.50534976514(76) \times 10^{-10}$ J
196	neutron mass energy equivalent in MeV	$\dot{\mathrm{E}}_n = 9.3956542194(48) \times 10^{2}$ MeV
197	neutron mass in u	$m_n/\mathrm{A_{mass}} = 1.00866491606(40)$ u
198	neutron molar mass	$\mathrm{M}_n = 1.00866491712(51) \times 10^{-3}$ kg/mol
199	neutron-proton magnetic moment ratio	$\mu_n/\mu_+ = -6.8497935(16) \times 10^{-1}$
200	neutron-proton mass difference	$m_\Delta = 2.30557461(67) \times 10^{-30}$ kg
201	neutron-proton mass diff energy equivalent	$\mathrm{E}_\Delta = 2.07214712(60) \times 10^{-13}$ J
202	neutron-proton mass diff energy equiv in MeV	$\dot{\mathrm{E}}_\Delta = 1.29333251(38)$ MeV
203	neutron proton mass difference in u	$m_\Delta/\mathrm{A_{mass}} = 1.38844948(40) \times 10^{-3}$ u
204	neutron relative atomic mass	$\mathrm{A_r(n)} = 1.00866491606(40)$
205	neutron to shielded proton mag mom ratio	$\mu_n/\mu_+' = -6.8499694(16) \times 10^{-1}$
206	Newtonian constant of gravitation	$G = 6.67430(15) \times 10^{-11}$ $\mathrm{m^3/s^2kg}$
207	Newtonian constant of gravitation over h-bar c	$G/\hbar c = 6.70883(15) \times 10^{-39}$ $\mathrm{c^4/GeV^2}$
208	nuclear magneton	$\mu_N = 5.0507837393(16) \times 10^{-27}$ J/T
209	nuclear magneton in eV/T	$\mu_N/e = 3.15245125417(98) \times 10^{-8}$ eV/T
210	nuclear magneton in inverse meter per tesla	$\mu_N/hc = 2.54262341009(79) \times 10^{-2}$ 1/m T
211	nuclear magneton in K/T	$\mu_N/k_B = 3.6582677706(11) \times 10^{-4}$ K/T
212	nuclear magneton in MHz/T	$\mu_N/h = 7.6225932188(24)$ MHz/T
213	Planck constant	$h = 6.62607015 \times 10^{-34}$ J s
214	Planck constant in eV/Hz	$h/e = 4.135667696 \times 10^{-15}$ eV/Hz
215	Planck electric impedance	$Z_p = 2.99792458 \times 10^{1}$ Ω
216	Planck mass energy equivalent in GeV	$\dot{\mathrm{E}}_p = 1.220890(14) \times 10^{19}$ GeV
217	proton charge to mass quotient	$\mathrm{e}/m_+ = 9.5788331430(29) \times 10^{7}$ C/kg
218	proton Compton wavelength	$\lambda_+ = 1.32140985360(41) \times 10^{-15}$ m
219	proton g factor	$g_+ = 5.5856946893(16)$
220	proton gyromagnetic ratio	$\gamma_+ = 2.6752218708(11) \times 10^{8}$ 1/s T
221	proton gyromagnetic ratio in MHz/T	$\gamma_+ = 4.2577478461(18) \times 10^{1}$ MHz/T
222	proton magnetic moment	$\mu_+ = 1.41060679545(60) \times 10^{-26}$ J/T
223	proton mag mom to Bohr magneton ratio	$\mu_+/\mu_B = 1.52103220230(45) \times 10^{-3}$
224	proton mag mom to nuclear magneton ratio	$\mu_+/\mu_N = 2.79284734463(82)$
225	proton magnetic shielding correction	$\sigma_+' = 2.56715(41) \times 10^{-5}$
226	proton mass	$m_+ = 1.67262192595(52) \times 10^{-27}$ kg
227	proton mass energy equivalent	$\mathrm{E}_+ = 1.50327761802(47) \times 10^{-10}$ J
228	proton mass energy equivalent in MeV	$\dot{\mathrm{E}}_+ = 9.3827208943(29) \times 10^{2}$ MeV
229	proton mass in u	$m_+/\mathrm{A_{mass}} = 1.0072764665789(83)$ u
230	proton molar mass	$\mathrm{M}_+ = 1.00727646764(31) \times 10^{-3}$ kg/mol
231	proton-neutron magnetic moment ratio	$\mu_+/\mu_n = -1.45989802(34)$
232	proton relative atomic mass	$\mathrm{A_r(+)} = 1.0072764665789(83)$
233	quantum of circulation	$q_c = 3.6369475467(11) \times 10^{-4}$ $\mathrm{m^2/s}$
234	quantum of circulation times 2	$2q_c = 7.2738950934(22) \times 10^{-4}$ $\mathrm{m^2/s}$
235	reduced Compton wavelength natural unit of length	$\lambda\!\!\!^{-} = 3.8615926744(12) \times 10^{-13}$ m
236	reduced muon Compton wavelength	$\lambda_{\mu^-} = 1.867594306(42) \times 10^{-15}$ m
237	reduced neutron Compton wavelength	$\lambda_{n^-} = 2.1001941520(11) \times 10^{-16}$ m
238	reduced Planck constant atomic unit of action	$\hbar = 1.054571817 \times 10^{-34}$ J s

	natural unit of action	
239	reduced Planck constant in eV s	$\hbar/e = 6.582119569 \times 10^{-16}$ eV s
240	reduced Planck constant times c in MeV fm	$\hbar c = 1.973269804 \times 10^{2}$ MeV fm
241	reduced proton Compton wavelength	$\lambda_{+\_} = 2.10308910051(66) \times 10^{-16}$ m
242	reduced tau Compton wavelength	$\lambda_{\tau\_} = 1.110538(75) \times 10^{-16}$ m
243	Rydberg constant	$R_\infty = 1.0973731568157(12) \times 10^{7}$ 1/m
244	Rydberg constant times c in Hz	$R_\infty c = 3.2898419602500(36) \times 10^{15}$ Hz
245	Rydberg constant times hc in eV	$R_\infty hc = 1.3605693122990(15) \times 10^{1}$ eV
246	Rydberg constant times hc in J	$R_\infty hc = 2.1798723611030(24) \times 10^{-18}$ J
247	Sackur-Tetrode constant (1 K, 100 kPa)	$ST_0 = -1.15170753496(47)$
248	Sackur-Tetrode constant (1 K, 101.325 kPa)	$ST_1 = -1.16487052149(47)$
249	second radiation constant	$c_2 = 1.438776877 \times 10^{-2}$ m K
250	shielded helion gyromagnetic ratio	$\gamma_{\mathrm{he}}{}' = 2.0378946078(18) \times 10^{8}$ 1/s T
251	shielded helion gyromagnetic ratio in MHz/T	$\gamma_{\mathrm{he}}{}' = 3.2434100033(28) \times 10^{1}$ MHz/T
252	shielded helion magnetic moment	$\mu_{\mathrm{he}}{}' = -1.07455311035(93) \times 10^{-26}$ J/T
253	shielded helion mag mom to Bohr mag ratio	$\mu_{\mathrm{he}}{}'/\mu_B = -1.15867149457(94) \times 10^{-3}$
254	shielded helion mag mom to nuclear mag ratio	$\mu_{\mathrm{he}}{}'/\mu_N = -2.1274977624(17)$
255	shielded helion to proton mag mom ratio	$\mu_{\mathrm{he}}{}'/\mu_+ = -7.6176657721(66) \times 10^{-1}$
256	shielded helion to shielded proton mag mom ratio	$\mu_{\mathrm{he}}{}'/\mu_+{}' = -7.617861334(31) \times 10^{-1}$
257	shielded proton gyromagnetic ratio	$\gamma_+{}' = 2.675153194(11) \times 10^{8}$ 1/s T
258	shielded proton gyromagnetic ratio in MHz/T	$\gamma_+{}' = 4.257638543(17) \times 10^{1}$ MHz/T
259	shielded proton magnetic moment	$\mu_+{}' = 1.4105705830(58) \times 10^{-26}$ J/T
260	shielded proton mag mom to Bohr mag ratio	$\mu_+{}'/\mu_B = 1.5209931551(62) \times 10^{-3}$
261	shielded proton mag mom to nuclear mag ratio	$\mu_+{}'/\mu_N = 2.792775648(11)$
262	shielding difference of d and p in HD	$\sigma_{\mathrm{dp}} = 1.98770(10) \times 10^{-8}$
263	shielding difference of t and p in HT	$\sigma_{\mathrm{tp}} = 2.39450(20) \times 10^{-8}$
264	speed of light in vacuum	$c = 2.99792458 \times 10^{8}$ m/s
	natural unit of velocity	
265	Stefan-Boltzmann constant	$\sigma = 5.670374419 \times 10^{-8}$ W/m^{2}K^4
266	tau Compton wavelength	$\lambda_\tau = 6.97771(47) \times 10^{-16}$ m
267	tau energy equivalent	$E_\tau = 1.77686(12) \times 10^{3}$ MeV
268	tau mass	$m_\tau = 3.16754(21) \times 10^{-27}$ kg
269	tau mass energy equivalent	$E_\tau = 2.84684(19) \times 10^{-10}$ J
270	tau mass in u	$m_\tau/A_{\mathrm{mass}} = 1.90754(13)$ u
271	tau molar mass	$M_\tau = 1.90754(13) \times 10^{-3}$ kg/mol
272	Thomson cross section	$\sigma_e = 6.6524587051(62) \times 10^{-29}$ m^2
273	triton g factor	$g_{\mathrm{tri}} = 5.957924930(12)$
274	triton magnetic moment	$\mu_{\mathrm{tri}} = 1.5046095178(30) \times 10^{-26}$ J/T
275	triton magnetic moment to Bohr mag ratio	$\mu_{\mathrm{tri}}/\mu_B = 1.6223936648(32) \times 10^{-3}$
276	triton magnetic moment to nuclear mag ratio	$\mu_{\mathrm{tri}}/\mu_N = 2.9789624650(59)$
277	triton mass	$m_{\mathrm{tri}} = 5.0073567512(16) \times 10^{-27}$ kg
278	triton mass energy equivalent	$E_{\mathrm{tri}} = 4.5003878119(14) \times 10^{-10}$ J
279	triton mass energy equivalent in MeV	$E_{\mathrm{tri}} = 2.80892113668(88) \times 10^{3}$ MeV
280	triton mass in u	$m_{\mathrm{tri}}/A_{\mathrm{mass}} = 3.01550071597(10)$ u
281	triton molar mass	$M_{\mathrm{tri}} = 3.01550071913(94) \times 10^{-3}$ kg/mol
282	triton relative atomic mass	$A_r(\mathrm{tri}) = 3.01550071597(10)$
283	triton to proton magnetic moment ratio	$\mu_{\mathrm{tri}}/\mu_+ = 1.0666399189(21)$
284	vacuum electric permittivity	$\varepsilon_0 = 8.8541878188(14) \times 10^{-12}$ F/m
285	vacuum magnetic permeability	$\mu_0 = 1.25663706127(20) \times 10^{-6}$ N/A^2
286	von Klitzing constant	$R_K = 2.581280745 \times 10^{4}$ Ω
	conventional value of the von Klitzing constant	
287	Wien frequency displacement law constant	$b' = 5.878925757 \times 10^{10}$ Hz/K
288	Wien wavelength displacement law constant	$b = 2.897771955 \times 10^{-3}$ m K

The constants of Nature do more than set the magnitudes of the actions of physics: they ensure that the atomic units—second, meter, coulomb, kelvin, kilogram—interact without contradiction. They enforce coherence across different domains of measurement. Their values reflect a deeper geometric structure holding the whole system together.

What follows is a detailed investigation into whether these 288 constants—the empirical skeleton of reality—are unified by a single underlying geometry.