Part II: Structural Rules of the Constants

Chapter 3: From Measurement to Manifold

Before explaining how the Transform Dictionary—the map of all 288 constants—was discovered, we should first look at a slice of it, so that its pattern can be seen directly, even before the symbols are fully understood.

To make this pattern visible, we display its structure explicitly. Every constant is constructed algebraically from Planck boundaries, partitioning and connecting according to the same two rules. The result is a grid—8 columns of 36 equations each—288 expressions governed by a single syntax. These algebraic-geometric connections are structural expressions of a grammar of transformation.

What follows is one vertical slice of the full grid—a single column composed of 36 algebraic-geometric expressions, each characterizing a constant of Nature. Don't worry about decoding each symbol just yet. What matters is the structure: the repeating pattern, the shared syntax, the consistent ingredients. The invitation is simply to look closely enough that the order becomes visible.

symbol/name legend

$\dot{\mathrm{E}}_p$ = Planck mass energy equivalent in GeV
R_K = von Klitzing constant
Z_0 = characteristic impedance of vacuum
Z_p = Planck electric impedance
$\mathrm{Hz} : \frac{1}{\mathrm{m}}$ = hertz-inverse meter relationship
$\mathrm{A}_{\mathrm{vel}}$ = atomic unit of velocity
$\mathrm{A}_{\mathrm{mom}}$ = atomic unit of momentum
$\mathcal{N}_{\mathrm{mom}}$ = natural unit of momentum
$\dot{\mathcal{N}}_{\mathrm{mom}}$ = natural unit of momentum in MeV/c
$\mathrm{E}_{\mathrm{A_{mass}}}$ = atomic mass constant energy equivalent in MeV
$\mathrm{kg} : \mathrm{J}$ = kilogram − joule relationship
$\mathrm{J} : \mathrm{kg}$ = joule-kilogram relationship
$\mathrm{A}_{\mathrm{mass}} : \mathrm{J}$ = atomic mass unit-joule relationship
$\mathrm{J} : \mathrm{A}_{\mathrm{mass}}$ = joule-atomic mass unit relationship
$\mathrm{kg} : \mathrm{eV}$ = kilogram-electron volt relationship
$\mathrm{eV} : \mathrm{kg}$ = electron volt-kilogram relationship
E_Δ = neutron-proton mass difference energy equivalent
$\dot{\mathrm{E}}_\Delta$ = neutron-proton mass diff energy equivalent in MeV
$\dot{\mathrm{E}}_e$ = natural unit of energy in MeV
$\dot{\mathrm{E}}_\mu$ = muon mass energy equivalent in MeV
$\dot{\mathrm{E}}_+$ = proton mass energy equivalent in MeV
$\dot{\mathrm{E}}_n$ = neutron mass energy equivalent in MeV
$\dot{\mathrm{E}}_\tau$ = tau mass energy equivalent in MeV
$\dot{\mathrm{E}}_{\mathrm{de}}$ = deuteron mass energy equivalent in MeV
$\dot{\mathrm{E}}_{\mathrm{he}}$ = helion mass energy equivalent in MeV
$\dot{\mathrm{E}}_{\mathrm{tri}}$ = triton mass energy equivalent in MeV
$\dot{\mathrm{E}}_\alpha$ = alpha particle mass energy equivalent in MeV
E_e = electron mass energy equivalent
E_μ = muon mass energy equivalent
E_+ = proton mass energy equivalent
E_n = neutron mass energy equivalent
E_τ = tau mass energy equivalent
E_{de} = deuteron mass energy equivalent
E_{he} = helion mass energy equivalent
$\mathrm{E}_{\mathrm{tri}}$ = triton mass energy equivalent
E_α = alpha particle mass energy equivalent

t_p = the Planck time, l_p = the Planck length, q_p = the Planck charge, T_p = the Planck temperature, and m_p = the Planck mass.

$$\dot{\mathrm{E}}_p = \frac{\mathrm{joule}}{\mathrm{GeV}}\left(\frac{{l_p}^2\, m_p}{{t_p}^2}\right)\left(1 - \frac{1}{2} Re\left(i^{i^{\cdot^{\cdot}}}\right)\text{ж}_3\,\text{ж}_4\,\boxtimes\right)$$

$$R_K = \frac{2\pi}{{\text{ж}_1}^2}\left(\frac{{l_p}^2\, m_p}{t_p\, {q_p}^2}\right)\left(1 + \frac{1}{2} Re\left(i^{i^{\cdot^{\cdot}}}\right)\text{ж}_3\,\text{ж}_4\,\boxtimes\right)$$

$$Z_0 = 4\pi\left(\frac{{l_p}^2 m_p}{t_p\, {q_p}^2}\right)\left(1 + \frac{1}{2} Re\left(i^{i^{\cdot^{\cdot}}}\right)\text{ж}_3\,\text{ж}_4\,\boxtimes\right)$$

$$Z_p = \left(\frac{{l_p}^2 m_p}{t_p\, {q_p}^2}\right)\left(1 + \frac{1}{2} Re\left(i^{i^{\cdot^{\cdot}}}\right)\text{ж}_3\,\text{ж}_4\,\boxtimes\right)$$

$$\mathrm{Hz} \vdots \frac{1}{\mathrm{m}} = \frac{1}{\mathrm{second}}\left(\frac{t_p}{l_p}\right)\left(1 + \frac{1}{2} Im\left(i^{i^{\cdot^{\cdot}}}\right)\text{ж}_3\,\text{ж}_4\,\boxtimes\right)$$

$$\mathrm{A_{vel}} = {\text{ж}_1}^2\left(\frac{l_p}{t_p}\right)\left(1 - \frac{1}{2} Im\left(i^{i^{\cdot^{\cdot}}}\right)\text{ж}_3\,\text{ж}_4\,\boxtimes\right)$$

$$\mathrm{A_{mom}} = {\text{ж}_1}^2\left(\frac{l_p\, m_e}{t_p}\right)\left(1 - \frac{1}{2} Im\left(i^{i^{\cdot^{\cdot}}}\right)\text{ж}_3\,\text{ж}_4\,\boxtimes\right)$$

$$\mathcal{N}_{\mathrm{mom}} = \left(\frac{l_p\, m_e}{t_p}\right)\left(1 - \frac{1}{2} Im\left(i^{i^{\cdot^{\cdot}}}\right)\text{ж}_3\,\text{ж}_4\,\boxtimes\right)$$

$$\mathrm{eV} \vdots \mathrm{kg} = \mathrm{eV}\left(\frac{{t_p}^2}{{l_p}^2}\right)\left(1 + Im\left(i^{i^{\cdot^{\cdot}}}\right)\text{ж}_3\,\text{ж}_4\,\boxtimes\right)$$

$$\mathrm{J} \vdots \mathrm{kg} = \mathrm{joule}\left(\frac{{t_p}^2}{{l_p}^2}\right)\left(1 + Im\left(i^{i^{\cdot^{\cdot}}}\right)\text{ж}_3\,\text{ж}_4\,\boxtimes\right)$$

$$\mathrm{J} \vdots \mathrm{A_{mass}} = \frac{\mathrm{joule}}{\mathrm{A_{mass}}}\left(\frac{{t_p}^2}{{l_p}^2}\right)\left(1 + Im\left(i^{i^{\cdot^{\cdot}}}\right)\text{ж}_3\,\text{ж}_4\,\boxtimes\right)$$

$$\mathrm{A_{mass}} \vdots \mathrm{J} = \mathrm{A_{mass}}\left(\frac{{l_p}^2}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{\cdot^{\cdot}}}\right)\text{ж}_3\,\text{ж}_4\,\boxtimes\right)$$

$$\dot{\mathcal{N}}_{\text{mom}} = \frac{\text{joule second}}{\text{MeV meter}} \left(\frac{{l_p}^2 \, m_e}{{t_p}^2} \right) \left(1 - Im \left(i^{i^{i^{\cdot^{\cdot}}}} \right) ж_3 \, ж_4 \, \boxtimes \right)$$

$$\dot{\text{E}}_{\text{A}_{\text{mass}}} = \frac{\text{A}_{\text{mass}}}{\text{MeV}} \left(\frac{{l_p}^2}{{t_p}^2} \right) \left(1 - Im \left(i^{i^{i^{\cdot^{\cdot}}}} \right) ж_3 \, ж_4 \, \boxtimes \right)$$

$$\text{kg} \vdots \text{J} = \text{kilogram} \left(\frac{{l_p}^2}{{t_p}^2} \right) \left(1 - Im \left(i^{i^{i^{\cdot^{\cdot}}}} \right) ж_3 \, ж_4 \, \boxtimes \right)$$

$$\text{kg} \vdots \text{eV} = \frac{\text{joule kilogram}}{\text{eV}} \left(\frac{{l_p}^2}{{t_p}^2} \right) \left(1 - Im \left(i^{i^{i^{\cdot^{\cdot}}}} \right) ж_3 \, ж_4 \, \boxtimes \right)$$

$$\text{E}_{\Delta} = \left(m_n - m_+ \right) \left(\frac{{l_p}^2}{{t_p}^2} \right) \left(1 - Im \left(i^{i^{i^{\cdot^{\cdot}}}} \right) ж_3 \, ж_4 \, \boxtimes \right)$$

$$\dot{\text{E}}_{\Delta} = \frac{\text{joule}}{\text{MeV}} \left(m_n - m_+ \right) \left(\frac{{l_p}^2}{{t_p}^2} \right) \left(1 - Im \left(i^{i^{i^{\cdot^{\cdot}}}} \right) ж_3 \, ж_4 \, \boxtimes \right)$$

$$\dot{\text{E}}_e = \frac{\text{joule}}{\text{MeV}} \left(\frac{{l_p}^2 \, m_e}{{t_p}^2} \right) \left(1 - Im \left(i^{i^{i^{\cdot^{\cdot}}}} \right) ж_3 \, ж_4 \, \boxtimes \right)$$

$$\dot{\text{E}}_{\mu} = \frac{\text{joule}}{\text{MeV}} \left(\frac{{l_p}^2 \, m_{\mu}}{{t_p}^2} \right) \left(1 - Im \left(i^{i^{i^{\cdot^{\cdot}}}} \right) ж_3 \, ж_4 \, \boxtimes \right)$$

$$\dot{\text{E}}_+ = \frac{\text{joule}}{\text{MeV}} \left(\frac{{l_p}^2 \, m_+}{{t_p}^2} \right) \left(1 - Im \left(i^{i^{i^{\cdot^{\cdot}}}} \right) ж_3 \, ж_4 \, \boxtimes \right)$$

$$\dot{\text{E}}_n = \frac{\text{joule}}{\text{MeV}} \left(\frac{{l_p}^2 \, m_n}{{t_p}^2} \right) \left(1 - Im \left(i^{i^{i^{\cdot^{\cdot}}}} \right) ж_3 \, ж_4 \, \boxtimes \right)$$

$$\dot{\text{E}}_{\tau} = \frac{\text{joule}}{\text{MeV}} \left(\frac{{l_p}^2 \, m_{\tau}}{{t_p}^2} \right) \left(1 - Im \left(i^{i^{i^{\cdot^{\cdot}}}} \right) ж_3 \, ж_4 \, \boxtimes \right)$$

$$\dot{\text{E}}_{\text{de}} = \frac{\text{joule}}{\text{MeV}} \left(\frac{{l_p}^2 \, m_{\text{de}}}{{t_p}^2} \right) \left(1 - Im \left(i^{i^{i^{\cdot^{\cdot}}}} \right) ж_3 \, ж_4 \, \boxtimes \right)$$

$$\dot{\mathrm{E}}_{\mathrm{he}} = \frac{\mathrm{joule}}{\mathrm{MeV}}\left(\frac{{l_p}^2\, m_{\mathrm{he}}}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}\right) \text{ж}_3\, \text{ж}_4 \boxtimes \right)$$

$$\dot{\mathrm{E}}_{\mathrm{tr\imath}} = \frac{\mathrm{joule}}{\mathrm{MeV}}\left(\frac{{l_p}^2\, m_{\mathrm{tri}}}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}\right) \text{ж}_3\, \text{ж}_4 \boxtimes \right)$$

$$\dot{\mathrm{E}}_{\alpha} = \frac{\mathrm{joule}}{\mathrm{MeV}}\left(\frac{{l_p}^2\, m_{\alpha}}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}\right) \text{ж}_3\, \text{ж}_4 \boxtimes \right)$$

$$\mathrm{E}_{e} = \left(\frac{{l_p}^2\, m_{e}}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}\right) \text{ж}_3\, \text{ж}_4 \boxtimes \right)$$

$$\mathrm{E}_{\mu} = \left(\frac{{l_p}^2\, m_{\mu}}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}\right) \text{ж}_3\, \text{ж}_4 \boxtimes \right)$$

$$\mathrm{E}_{+} = \left(\frac{{l_p}^2\, m_{+}}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}\right) \text{ж}_3\, \text{ж}_4 \boxtimes \right)$$

$$\mathrm{E}_{n} = \left(\frac{{l_p}^2\, m_{n}}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}\right) \text{ж}_3\, \text{ж}_4 \boxtimes \right)$$

$$\mathrm{E}_{\tau} = \left(\frac{{l_p}^2\, m_{\tau}}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}\right) \text{ж}_3\, \text{ж}_4 \boxtimes \right)$$

$$\mathrm{E}_{\mathrm{de}} = \left(\frac{{l_p}^2\, m_{\mathrm{de}}}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}\right) \text{ж}_3\, \text{ж}_4 \boxtimes \right)$$

$$\mathrm{E}_{\mathrm{he}} = \left(\frac{{l_p}^2\, m_{\mathrm{he}}}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}\right) \text{ж}_3\, \text{ж}_4 \boxtimes \right)$$

$$\mathrm{E}_{\mathrm{tri}} = \left(\frac{{l_p}^2\, m_{\mathrm{tri}}}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}\right) \text{ж}_3\, \text{ж}_4 \boxtimes \right)$$

$$\mathrm{E}_{\alpha} = \left(\frac{{l_p}^2\, m_{\alpha}}{{t_p}^2}\right)\left(1 - Im\left(i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}\right) \text{ж}_3\, \text{ж}_4 \boxtimes \right)$$

Beneath each of these 36 constants of Nature lies an iterated complex exponential: $i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}$. This is known as the infinite power tower of i—a recursively defined exponential structure in which i (the imaginary unit) is repeatedly raised to the power of itself.

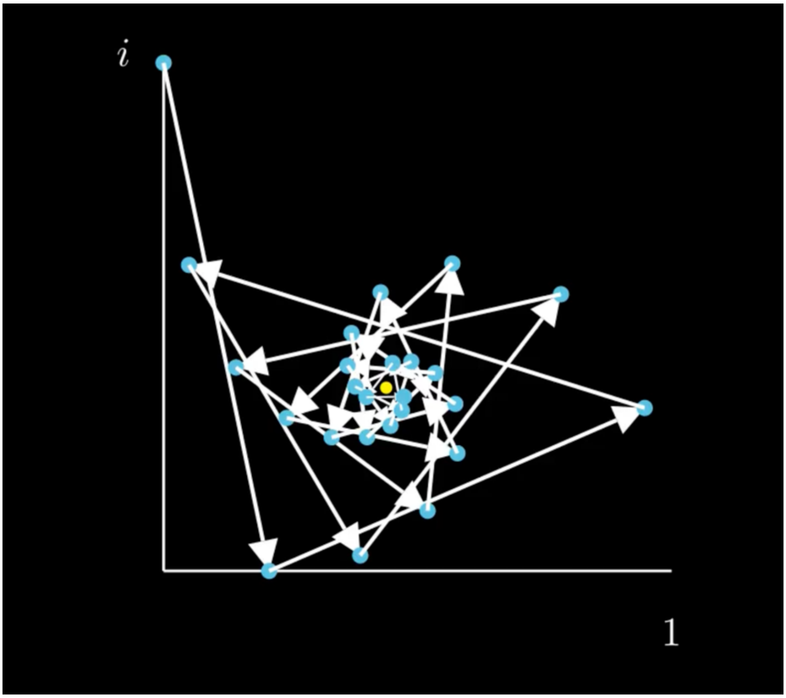

Figure 1: The power tower of i traces interleaved spiral strands in the complex plane that wind inward toward a single fixed point—the yellow point—located between the real and imaginary axes.[36]

The successive iterates of the power tower of i trace out a spiraling structure in the complex plane that orbits and converges on a single fixed point—hovering between the real and imaginary axes. When iterated infinitely, this power tower stabilizes to that fixed point. That this same iterative geometry reappears again and again beneath the surface of each constant in this column suggests that this sector of Nature's architecture is patterned by the logic of complex iteration.

The infinite power tower of i therefore becomes a structural clue. It links what otherwise appear to be disparate constants by a shared recursive process. The same geometric ingredient is recurring across measured quantities.

At first glance, the constants of Nature are an eclectic catalog of quantities. Some have dimensions of mass, charge, or energy, while others are dimensionless—representing gyromagnetic ratios, or magnetic moment ratios. Some have enormous exponents, others miniscule ones.

But when we reframe these constants in terms of Planck boundaries, a different picture emerges. The constants of Nature can be organized as constructs build from five geometric limits—the Planck time, Planck length, Planck charge, Planck temperature, and Planck mass—and their bases, woven into one coherent system via two fundamental structural rules. We will develop this in detail later. The first is a binomial combinatorial rule for coherently integrating transformations about Planck boundaries. It enforces that every transformation is composed of two parts—an external part and an internal one.

the binomial constructor

$$\mathrm{A_{ext}}\,\mathrm{B_{ext}}\left(1+\mathrm{A_{int}}\boxtimes\right) \qquad \boxtimes=\frac{l_p m_p}{{q_p}^2}\times\frac{\text{coulomb}^2}{\text{meter kilogram}}$$

Where $\mathrm{A_{ext}}=$ the external geometric action, $\mathrm{B_{ext}}=$ the external arrangement of atomic bases or Planck boundaries, $\mathrm{A_{int}}=$ the internal geometric action, and $\boxtimes=$ the internal inversion boundary—composed of normalized Planck boundaries: l_p/meter, $m_p/\text{kilogram}$, and $q_p/\text{coulomb}$ squared.

The second rule governs how this system partitions within the Planck mass gap—how it subdivides while remaining dynamically whole.

the hyperbolic partition equation

$$\frac{1}{x}+x+\frac{x^3}{2\pi}=\left(i^i\right)^{-\frac{4\pi}{8}}-\frac{m_p}{\text{kg}}$$

Where $\pi=$ Archimedes' constant, $i=$ the imaginary unit, i^i is evaluated on the principal branch, and $m_p/\text{kilogram}=$ the normalized Planck mass.

This defines a monic quartic in x whose four roots—$ж_1$, $ж_2$, $ж_3$, $ж_4$—encode how this hyperbolic structure partitions within the Planck mass gap.[37]

Together, these two rules define a proposed logic space—a constraint architecture—within which the constants of Nature can be represented as structural invariants. This architecture maps how reality self-organizes from the inside out.

These rules organize all 288 constants of Nature into one coherent geometric language. Each constant falls into place as a precise expression of the same underlying logic: five atomic bases and Planck boundaries governed by two symmetry-based rules. The constants don't just reflect Nature—they reveal the structural logic by which Nature builds itself. Under that logic the constants become more than a list of measured facts. They become a possible grammar: a syntax of transformation woven through the quantitative features of the Universe.

In the following chapters we will show that these two rules reproduce the measured magnitudes and dimensions of all 288 CODATA constants. Under these rules, each constant of Nature is represented as a bi-part manifold transformation.

What follows is a step-by-step reconstruction of that grammar. We will show how each expression encodes a specific measured value—and how the recurring components function as geometric operations. As we do, we will trace the components of those constants to the algebraic and geometric powers conferred by the hyperbolic figure-eight knot. This will lead us to ask whether that geometry can sustain the algebraic skeleton of the Dirac equation.

After identifying a base geometry for this system, we will use the structural features of that geometry to justify the two rules that exposed it—the binomial constructor and the hyperbolic partition equation. Then we will discover that the set of possible transition states available to that base geometry defines the 24-dimensional unit ball B^{24}. This framing geometry gives the five Planck boundaries precise, structured geometric homes.

Finally, we will explore what it means—ontologically, mathematically, and physically—to access a geometric structure that governs how the world transitions from one state to the next while remaining coherently whole.