Chapter 6: The partition equation emerges

> "There is a most profound and beautiful question associated with the observed coupling constant, *e*—the amplitude for a real electron to emit or absorb a real photon. It is a simple number that has been experimentally determined to be close to 0.08542455. (My physicist friends won't recognize this number, because they like to remember it as the inverse of its square: about 137.03597 with about an uncertainty of about 2 in the last decimal place. It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.) Immediately you would like to know where this number for a coupling comes from: is it related to π or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!"
> —Richard Feynman[43]

> "When I die, my first question to the Devil will be: What is the meaning of the fine-structure constant?"
> —Attributed to Wolfgang Pauli[44]

For years, I chased a single mystery: What does the fine-structure constant mean? Not just because it governs the strength of electromagnetism, or because Dirac and Feynman thought it was the simplest keyhole into the rules of reality—being dimensionless—but because it shows up more than 100 times in the constants of Nature—binding Planck boundaries together. Behind that prolific number was a riddle to unlock—a riddle directly related to how the Planck boundaries partition.

In trying to uncover the rule that governs how Planck boundaries internally partition while obeying the binomial constructor, I read every paper I could find about the fine-structure constant; every attempt to derive it from geometric principles, every speculative connection between known constants and dimensionless ratios. Many had tried. Some had found numerical approximations, the best of which landed five digits. Rarely more. Wyler's constant was among the champions of simple combinations that reproduce the fine-structure constant to high accuracy—5 digits—but it offered no pathway to understanding—no invitation to meaning.[45]

$$W_{\mathrm{Wy}} = \frac{9}{8\,\pi^4}\left(\frac{\pi^5}{2^4 \cdot 5!}\right)^{1/4}$$

$\alpha = 7.2973525693(11) \times 10^{-3}$ CODATA 2014
$W_{\mathrm{Wy}} = 7.2973481300318\ldots \times 10^{-3}$ Wyler's constant

Then, on March 21, 2019, an email arrived from Phillip Preetz. He knew I was searching and pointed me to a blog post[46] by Hans de Vries. In it, de Vries observed something remarkable: an expression involving only e and π that approximates the square root of the fine-structure constant.[47] That claim caught my attention. But when I checked the math…

$$\frac{1}{\sqrt{\beta}} + \sqrt{\beta} + \frac{\sqrt{\beta}^{3}}{2\pi} = e^{\pi^2/4}$$

$\alpha = 7.2973525693(11) \times 10^{-3}$ CODATA 2014
$\beta = 7.2973525456205\ldots \times 10^{-3}$ Hans de Vries number

Where $\sqrt{\alpha}$ = the number Feynman was referring to: 0.08542455.

The match was stunning: eight digits correct—making his expression a thousand times more precise than Wyler's constant.

Even more striking, it did it with a *formula* built only from e and π. This was not merely a better fit. It was an equation of a geometric form—a shape that could be investigated.

De Vries's approximation wasn't just better than Wyler's; it provided a structure, revealing a form with symmetries and roots. But should I investigate? Or dismiss it as a coincidence? After all, the number we are talking about generating with this geometric rule is not the *correct* number. It may be the closest anyone had ever come to the correct number using simple algebraic expressions, but it does not faithfully reproduce the measured number. Specifically, its value is greater than twenty-one σ off—falling outside the error bars of the measured number by more than twenty-one times the size of those error bars—five σ is the conventional "discovery threshold" in particle physics, where σ = one standard deviation.

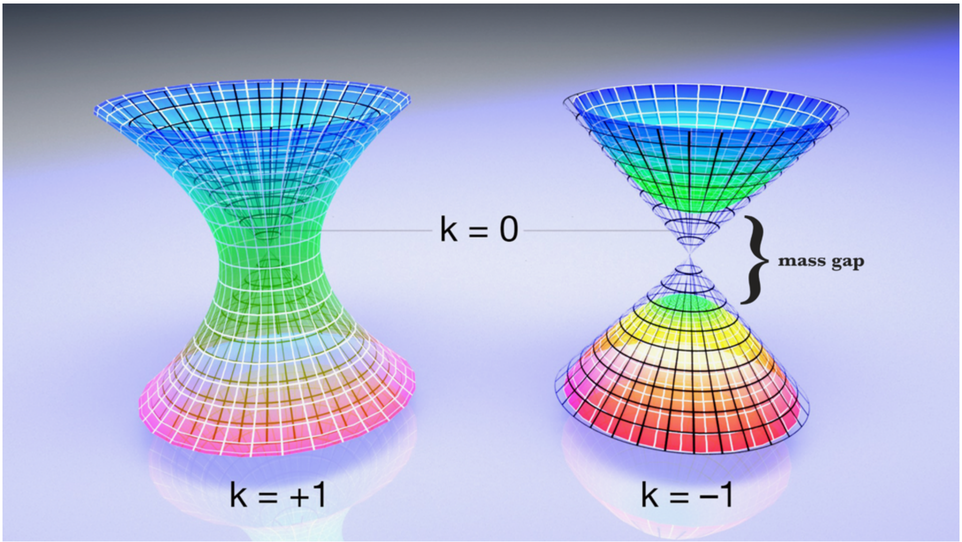
The simple geometric structure of de Vries's equation made it impossible to ignore. I needed an equation with exactly that kind of geometric power, but I needed it to accurately characterize the fine-structure constant—so I could use it to get to know *that* number. So, I asked: What would it take to correct de Vries' equation? How far off is it quantitatively?

To find out, I subtracted a small correction term, δ, and then plugged in the measured value of α to solve for δ:

$$\frac{1}{\sqrt{\alpha}} + \sqrt{\alpha} + \frac{\sqrt{\alpha}^3}{2\pi} = e^{\pi^2/4} - \delta \qquad \delta = 1.65(13) \times 10^{-8}$$

There is only one Planck boundary possessing an exponent of -8 —the Planck mass. So, I looked up the digits of the Planck mass. And there it was. The correction term—what the equation "missed"—was on the scale of the normalized Planck mass.

$$\frac{m_p}{\text{kg}} = 2.17647(24) \times 10^{-8} \qquad \text{CODATA 2014}$$



Left: the one-sheeted hyperboloid surrounding the double-cone. Right: the two-sheeted hyperboloid inside the double cone—illustrating the natural 'mass gap' of hyperbolic geometry.

The appearance of a normalized Planck mass scale correction suggested that the partition rule might be tied to a hyperbolic gap structure. Hyperbolic geometries possess a natural feature known as a *mass gap*: the separation between the two sheets of their defining hyperboloid. A hyperbolic system with a normalized Planck mass gap is also the ideal partitioner of a Planck-bounded binomial construction. (See Appendix A for a discussion of how this geometric mass gap relates to the spectral mass gap of Yang-Mills theory.)

I rewrote de Vries' expression—centering it on $\sqrt{\alpha}$ instead of α—and set the gap to the normalized Planck mass. This produced the hyperbolic partition equation—the governing partition rule behind the dual-layer structure encoded in the constants of Nature.

the hyperbolic partition equation

$$\frac{1}{x} + x + \frac{x^3}{2\pi} = \left(i^i\right)^{-\frac{4\pi}{8}} - \frac{m_p}{\mathrm{kg}}$$

Where π = Archimedes' constant, i = the imaginary unit, $m_p/\mathrm{kilogram}$ = the normalized Planck mass, $e^{\pi^2/4} = \left(i^i\right)^{-\frac{4\pi}{8}}$, and $i^i = e^{-\pi/2}$ is the principal branch of the complex logarithm.

The structure of this equation—its interplay of reciprocal and cubic terms—suggested a story of inversion, balance, and emergence, characterizing the partition structure of a hyperbolic geometry with a mass gap equal to the normalized magnitude of the Planck mass.

If the binomial constructor enforces the cohesive union of two layers of Planck boundaries, this partition equation offers a rule for how those layers coherently subdivide, defining the internal structure that allows persistent, self-consistent subsystems to form.

Looking closer, we discover the rich algebraic-geometric structure of this equation. For example, the hyperbolic partition equation possesses 4 distinct solutions, or roots.

$$ж_1 = 0.0854245431533304\ldots$$
$$ж_2 = 3.66756753485501\ldots$$
$$ж_3 = -1.87649603900417\ldots + 4.06615262615972\ldots i$$
$$ж_4 = -1.87649603900417\ldots - 4.06615262615972\ldots i$$

The square of the first solution matches the measured fine-structure constant to 3.32σ (5.00σ is the standard threshold).

$\alpha = 7.2973525693(11) \times 10^{-3}$	CODATA 2014
$ж_1{}^2 = 7.2973525729557 \ldots \times 10^{-3}$	new number

But that's just the beginning. These four constants carry a remarkable algebraic-geometric structure. For example, their product, sum and quadrance define a tightly constrained root system balanced over a single inversion.

Their product is 2π, their sum is 0, and their quadrance—the sum of their squares—is -4π.

product	$ж_1\, ж_2\, ж_3\, ж_4 = 2\pi$
sum	$ж_1 + ж_2 + ж_3 + ж_4 = 0$
quadrance	$ж_1{}^2 + ж_2{}^2 + ж_3{}^2 + ж_4{}^2 = -4\pi$

The 1st component of that quadrance *is* the fine-structure constant $ж_1{}^2 = \alpha$, now revealed as one element of a highly ordered algebraic-geometric structure, accompanied by three sibling quadrance components: $ж_2{}^2$, $ж_3{}^2$, and $ж_4{}^2$.

Once these roots are visible, the next task is to ask which partial combinations of them can function as real scalar actions. Because $ж_3$ and $ж_4$ are complex conjugates, not every partial combination is admissible as a simple real transform. Coherent scalar actions must either use the real roots directly, pair the conjugate roots symmetrically, or pass through the polar data of the conjugate pair.

	2-part	**3-part**
products	$ж_1\, ж_2$	$ж_1\, ж_3\, ж_4$
	$ж_3\, ж_4$	$ж_2\, ж_3\, ж_4$
sums	$(ж_1 + ж_2)$	$(ж_1 + ж_3 + ж_4)$
	$(ж_1 - ж_2)$	$(ж_1 - ж_3 - ж_4)$
	$(ж_3 + ж_4)$	$(ж_2 + ж_3 + ж_4)$
	$(ж_3 - ж_4)$	$(ж_2 - ж_3 - ж_4)$
quadrances	$(ж_1{}^2 + ж_2{}^2)$	$(ж_1{}^2 + ж_3{}^2 + ж_4{}^2)$
	$(ж_1{}^2 - ж_2{}^2)$	$(ж_1{}^2 - ж_3{}^2 - ж_4{}^2)$
	$(ж_3{}^2 + ж_4{}^2)$	$(ж_2{}^2 + ж_3{}^2 + ж_4{}^2)$

$$(ж_3{}^2 - ж_4{}^2) \quad (ж_2{}^2 - ж_3{}^2 - ж_4{}^2)$$

$$\begin{matrix} ж_r{}^2 & ж_\theta{}^2 \\ ж_r{}^3 & ж_\theta{}^3 \\ ж_r{}^4 & ж_\theta{}^4 \end{matrix}$$

The 8 partition groups: where $ж_3 = ж_r\, e^{ж_\theta\, i}$, $ж_4 = ж_r\, e^{-ж_\theta\, i}$, hence $ж_3\, ж_4 = ж_r{}^2$, and $|ж_3| = |ж_3| = ж_r$.

This produces a finite family of simple partition actions: 2-part and 3-part products, sums, and quadrances, together with the polar radius and angle associated with the conjugate pair. These are the root-level actions that the constants of Nature will be checked against.

These eight decomposition groups represent the working family of simple real partition actions associated with the fine-structure constant. All other plural partial sets of these hyperbolic partition constants yield complex outputs, and therefore do not represent simple linear transforms of the system.

With these groups in hand, we can begin to examine the constants of Nature one by one—checking their algebraic structure against the hyperbolic partition equation's set of real root transformations.

Before doing so, let us first identify the invariants of this structure—the algebraic relations dictated by its roots.