

## Chapter 8: Closed-form Planck Boundaries

Before we can construct a world from five Planck boundaries—governed by two structural rules, the binomial constructor and the hyperbolic partition equation—we must have precise closed-form definitions for those boundaries.

We've already seen that the constants of Nature repeatedly point toward the five Planck boundaries:  $t_p$ ,  $l_p$ ,  $q_p$ ,  $T_p$ , and  $m_p$ . These are the boundaries of coherent atomic logic, appearing again and again within the ratios and transformations that make physics work. Yet despite their foundational role, these boundaries are not known with full precision.

That precision limit comes from the way the Planck boundaries are obtained. They are derived from the constants of Nature. Different combinations yield values that agree to roughly seven digits, then diverge. This divergence is a signal. It reveals a two-layer boundary structure.

If we are to build a unified system from Planck boundaries, we cannot rely on fuzzy inputs. If the Planck boundaries are truly structural, then their ultimate definitions cannot depend on which measurement pathway we choose. They should be defined, not just partially known. And that means they should be given precise closed-form definitions.

That became the mission: to hunt for closed forms that could serve as candidate definitions of the Planck boundaries themselves. Not approximations. Exact geometric actions that return each Planck boundary's value. If the Planck boundaries are real—if they are the structural anchors of Nature—then there must exist a geometry that pins them in place.

So, I began searching.

The search was algorithmic: convert each boundary into a dimensionless target, then look for simple closed forms—built from hyperbolic-geometric ingredients—that reproduce its digits. I sought every simple algebraic-geometric expression that characterized something modular or structurally meaningful. Anything composed of geometric elements— $e$ ,  $\pi$ , and functions like  $\sin(x)$ ,  $\cos(x)$ ,  $\sinh(x)$ , or  $\cosh(x)$ —that produced the reported digits of the Planck boundaries and could be made coherent as a set.

I used WolframAlpha to search for simple closed-form expressions that returned the experimentally observed values, and then systematically tested whether those expressions—when treated as exact—could reproduce the constants of Nature through the binomial constructor and hyperbolic partition logic.

Of course, precision alone wasn't enough. I was searching for five expressions that worked together—five closed forms that shared the same kind of internal logic, respected the same symmetry framework, and hinted at a single geometric ancestry.

So, my search was narrowed by two key constraints. Simplicity—expressions with cleaner algebraic structure were favored. And hyperbolic character—expressions whose elements appeared naturally in hyperbolic geometry. I then used the closed forms I found as exact inputs—building the constants of Nature from them to test whether they revealed any pattern between them.

To organize this search, we separate each Planck boundary, once normalized by its associated unit, into three numerical features: its partially known digit string, its base-ten scale, and the signed order of magnitude carried by that scale.

Let  $B_k$  denote the  $k^{\text{th}}$  Planck boundary:

$$\begin{aligned} B_0 &= t_p \\ B_1 &= l_p \\ B_2 &= q_p \\ B_3 &= T_p \\ B_4 &= m_p \end{aligned}$$

Each boundary is made dimensionless by measuring it relative to its associated unit:

$$\begin{aligned} u_0 &= \text{second} \\ u_1 &= \text{meter} \\ u_2 &= \text{coulomb} \\ u_3 &= \text{kelvin} \\ u_4 &= \text{kilogram} \end{aligned}$$

The numerical value of each normalized Planck boundary  $B_k/u_k$  can then be written in scientific notation as

$$\frac{B_k}{u_k} = \phi_k \times 10^{n_k}.$$

Here,  $\phi_k \in [1,10)$  is the significand: the partially known digit string. The factor 10 gives the base-ten scale. The integer  $n_k$  is the signed order of magnitude carried by that scale.

The known normalized values of the five Planck boundaries are:

$$\begin{aligned}
 m_p/\text{kg} &= 2.176434(24) \times 10^{-8} \\
 T_p/\text{K} &= 1.416784(16) \times 10^{32} \\
 q_p/\text{C} &= 1.8755459 \times 10^{-18} \\
 l_p/\text{m} &= 1.616255(18) \times 10^{-35} \\
 t_p/\text{s} &= 5.391247(60) \times 10^{-44}
 \end{aligned}$$

The corresponding digit strings and signed integer exponents are:

$$\begin{aligned}
 \phi_4 &= 2.176434(24) & n_4 &= -8 \\
 \phi_3 &= 1.416784(16) & n_3 &= +32 \\
 \phi_2 &= 1.8755459 & n_2 &= -18 \\
 \phi_1 &= 1.616255(18) & n_1 &= -35 \\
 \phi_0 &= 5.391247(60) & n_0 &= -44
 \end{aligned}$$

To probe this structure we ask: what dimensionless geometric action  $A_k$  must be multiplied by  $e^{\phi_k}$  to reproduce the signed exponent  $n_k$ ?

$$A_k e^{\phi_k} = n_k \quad \text{equivalently} \quad A_k = \frac{n_k}{e^{\phi_k}}.$$

This ties the partially known digit string to the signed order of magnitude carried by the decimal base.

To unveil each Planck boundary's connective geometric action  $A_k$ , I plugged in the respective  $\phi_k$  values, and searched for simple algebraic-geometric expressions that characterize the resulting signed numbers.

$$\begin{aligned}
 A_4 &= -8/e^{2.176434(24)} & \text{Planck mass} \\
 A_3 &= +32/e^{1.416784(16)} & \text{Planck temperature} \\
 A_2 &= -18/e^{1.8755459} & \text{Planck charge} \\
 A_1 &= -35/e^{1.616255(18)} & \text{Planck length} \\
 A_0 &= -44/e^{5.391247(60)} & \text{Planck time}
 \end{aligned}$$

This time, a matched set appeared. The search returned five simple, logically-related hyperbolic expressions—one for each Planck boundary. Each expression reproduces the corresponding CODATA value within its reported uncertainty, and because each is written in closed form, each candidate value can be evaluated to arbitrary precision.

### candidate closed-forms

$$\begin{aligned}
 \mathbf{5} \left( \frac{4\pi}{2} \right) \left( \cos \left( \frac{7}{5} \right) \right)^2 &= \frac{8}{e^{\Phi_4}} && \text{mass} \\
 2 \left( \frac{\mathbf{5}}{\sqrt{7}} \right)^2 \left( \cos \left( \frac{5}{2} i \right) \right)^2 \left( \cos \left( \frac{7}{5} \right) \right)^2 &= \frac{32}{e^{\Phi_3}} && \text{temperature} \\
 \left( \frac{\mathbf{5}}{\sqrt{7}} \right) \left( \frac{3^{-1/3}}{W_{We}} \right) &= \frac{18}{e^{\Phi_2}} && \text{charge} \\
 \left( \sinh \left( \sinh \left( \frac{1}{7} \right) \right) \right)^{-1} &= \frac{35}{e^{\Phi_1}} && \text{length} \\
 \left( \frac{4\pi}{4} \right) \left( \sinh \left( \frac{1}{4} \right) \right)^2 &= \frac{\mathbf{!5}}{e^{\Phi_0}} && \text{time}
 \end{aligned}$$

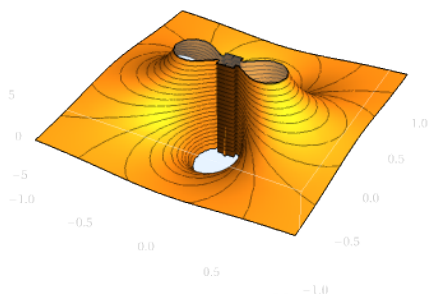
Where  $\cos(x)$  = the cosine function,  $\sinh(x)$  = the hyperbolic sine function,  $\pi$  = Archimedes' constant,  $i$  = the imaginary unit,  $W_{We}$  = the Weierstrass constant, and  $\mathbf{!5} = 44$ , with  $\mathbf{!}n$  = the derangement (subfactorial) function defined as  $\mathbf{!}n = n! \sum_{k=0}^n \frac{(-1)^k}{k!}$ .

coherent Planck constants	CODATA values	$\sigma = 1.00$ stan. dev.
$m_p = 2.1764268381757 \times 10^{-8}$ kg	$m_p = 2.176434(24) \times 10^{-8}$ kg	$\sigma = 0.33$
$T_p = 1.4167869859079 \times 10^{32}$ K	$T_p = 1.416784(16) \times 10^{32}$ K	$\sigma = 0.13$
$q_p = 1.8755459671396 \times 10^{-18}$ C	$q_p = 1.8755459 \times 10^{-18}$ C	$\sigma = 0.00$
$l_p = 1.6162591817564 \times 10^{-35}$ m	$l_p = 1.616255(18) \times 10^{-35}$ m	$\sigma = 0.22$
$t_p = 5.3912583683231 \times 10^{-44}$ s	$t_p = 5.391247(60) \times 10^{-44}$ s	$\sigma = 0.18$

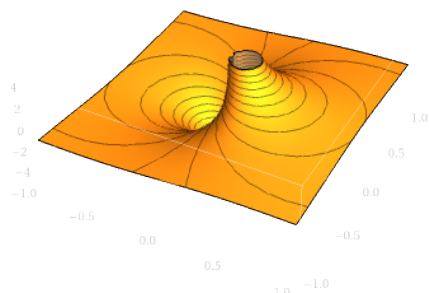
Five candidate boundary expressions—simple, modular, hyperbolic—emerge: each built from the language of geometry, each reproducing the CODATA value for a Planck boundary within the reported  $1\sigma$  uncertainty.

To visualize the structural behavior of these definitions, we extend their trigonometric or hyperbolic arguments into the complex plane.<sup>59</sup> The

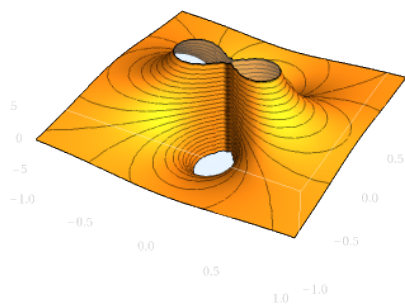
resulting surface plots allow us to diagnose the analytic character of the candidate boundary definitions and compare their shared geometric behavior.



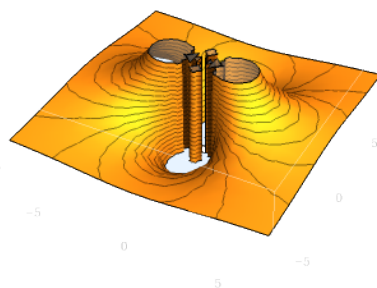
Planck time



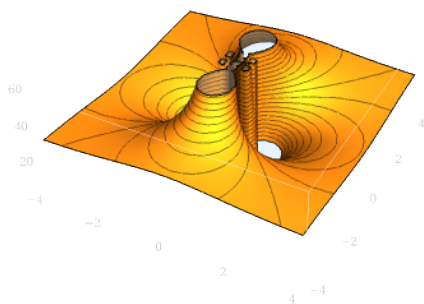
Planck length



Planck charge



Planck temperature



Planck mass

These surface plots expose strikingly common structure. Each surface is organized by lemniscate-like contours, with one branch ascending and another descending through the central region. The Planck length surface is shaped by a split lemniscate connection along its central spine,

while the Planck time, charge, temperature, and mass surfaces are composed of paired ascending and descending lemniscate connections. The Planck mass surface is oriented orthogonally to the other four, and the Planck charge surface has the simplest self-intersection.

Together, these expressions exhibit a shared coherence. They obey the same symmetry logic, draw from the geometric ingredients— $\cos(x)$ ,  $\sinh(x)$ ,  $\pi$ ,  $e$ ,  $i$ , and the Weierstrass constant  $W_{We}$ —capable of fitting together as parts of a single architectural scheme. This internal consistency was precisely what I had been searching for.

$$i^{-i} = \frac{s^4 W_{We}^4}{8} \quad e^\pi = \frac{s^8 W_{We}^8}{8^2} \quad s = \frac{\Gamma^2\left(\frac{1}{4}\right)}{\sqrt{2\pi}}$$

$$e^\pi = 4 L^8 W_{We}^8 \quad s = 2L$$

The Weierstrass constant— $W_{We}$ —is interwoven with  $\pi$ ,  $e$ ,  $i$ , through the arc length of the unit lemniscate— $s$ —the lemniscate constant— $L$ , and the gamma function  $\Gamma(x)$ . These relations place these candidate Planck boundary definitions within a unified web of classical geometric constants.

Of course, this does not mean these are the true definitions of the Planck boundaries. Not yet. At this stage, they are still hypotheses—tools in the toolkit. But they are simple and exact tools, from which we can now attempt to compose the constants of Nature.

To earn their place in the theory, these candidate definitions must do more than match the five Planck values. They must help reconstruct the constants of Nature as one organized system. Only then can the boundaries justify themselves structurally.

So, I began using these definitions of the Planck boundaries to deconstruct every constant of Nature—through the logic of the binomial constructor and the hyperbolic partition equation.

I had tried many possible sets of Planck-boundary definitions by this point, finding few clues to follow. But with this coherent set of definitions, a pattern began to emerge—one that, over the next couple of years, expanded to encompass all 288 constants of Nature.

With each new constant decoded the pattern became clearer—and more striking. The constants of Nature began to appear to be woven into a single structure. And the geometric components responsible for maintaining each boundary within that structure traced to a common origin.

Not only did this structure cohere; its geometric elements appeared to trace back to a very special geometry: the simplest possible arena constructible as a 3-manifold. The hunt for Planck boundaries—the boundaries built into the rules of physics—led us to the simplest geometric stage: a manifold capable of carrying structure, transformation, and persistence.

The next chapter examines the three constants of Nature that assemble the Dirac equation, and asks whether the geometry of the simplest constructible 3-manifold reveals itself in their structure.