

## Transform Dictionary

Every constant of Nature defines a transform about two sets of Planck boundaries. Each closed-form expression reproduces the correct magnitude and units of the associated constant. Collectively, these expressions use all eight types of transforms in the hyperbolic partition equation—defining the structure of the Dictionary’s columns.

Each entry includes a name, symbol,<sup>1</sup> and a bi-part equation with four signatures each. The first two signatures define the external geometric action and the external boundaries that action is taking place on. The last two define the internal geometric action and the internal inversion boundaries—a fixed feature, constructively shared by all constants.

Every equation in this Dictionary is followed by a paragraph labelling each algebraic participant in that equation. This is followed by the number and dimension predicted by this equation, which is then compared with the CODATA 2022 & 2018 listed values—in terms of their own error bars, where  $\sigma = 1.00$  reflects an error equal to the size of the error bars.

These are followed by a measure of the change in precision—in digits—the precision change in digits that going from the CODATA 2018 value to the CODATA 2022 value represents.  $\Delta_{\text{precision}} = 1.00$  represents an increase of 1 digit in precision. And finally, by a measure of how much the CODATA 2022 number represents a change to the old number, compared to its error bars.  $\Delta_{\text{scaled}} = 1.00$  means the new number swung by the same size as the error bars of the old number.

This is followed by any relevant notes, such as “CODATA also lists this constant as...”.

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<sup>1</sup> All names with the word *relationship* in them have been given symbols meant to convey the constant’s name. For example, the inverse meter-kelvin relationship is given the symbol  $\frac{1}{m} : K$ . Where  $:$  represents the word *relationship*. This is my symbol convention—not CODATA’s.

## Legend

$t_p = 5.39125836832313 \dots \times 10^{-44}$ s	Planck time
$l_p = 1.61625918175645 \dots \times 10^{-35}$ m	Planck length
$q_p = 1.87554596713962 \dots \times 10^{-18}$ C	Planck charge
$T_p = 1.41678698590795 \dots \times 10^{32}$ K	Planck temperature
$m_p = 2.17642683817579 \dots \times 10^{-8}$ kg	Planck mass
$\boxtimes = 0.999999199973626 \dots \times 10^{-7}$ dimensionless	inversion boundary
<b>5</b>	number of dimensions
<b>7</b>	break in scale symmetry
$\mathfrak{K}_1 = 0.0854245431533304 \dots$	1 <sup>st</sup> hyperbolic partition constant
$\mathfrak{K}_2 = 3.66756753485501 \dots$	2 <sup>nd</sup> hyperbolic partition constant
$\mathfrak{K}_3 = -1.87649603900417 \dots + 4.06615262615972 \dots i$	3 <sup>rd</sup> hpc
$\mathfrak{K}_4 = -1.87649603900417 \dots - 4.06615262615972 \dots i$	4 <sup>th</sup> hpc
$\mathfrak{K}_r = 4.47826244916751 \dots$	hyperbolic radius constant
$\mathfrak{K}_\theta = 2.00316562310924 \dots$	hyperbolic radian constant
$G_{Gi} = 1.01494160640965 \dots$	Gieseking's constant
$V_{fe} = 2.02988321281930 \dots$	figure-eight knot hyperbolic volume
$K = 0.9159655941772190 \dots$	Catalan's constant
$C_d = 0.291560904030818 \dots$	domino tiling constant
$D_d = 1.79162281206959 \dots$	dimer constant
$\omega_1 = 0.764977018528596 \dots + 1.32497062714087 \dots i$	omega_1 constant
$\omega_2 = 1.529954037057192 \dots$	omega_2 constant
$i^{i^{i^{\dots}}} = 0.438282936727032 \dots + 0.360592471871385 \dots i$	power tower of $i$
$i = (-1)^{1/2}$	imaginary unit
$\varphi_i = (-1)^{1/3}$	imaginary golden ratio
$\varphi = 1.61803398874989$	golden ratio
$\pi = 3.14159265358979 \dots$	Archimedes' constant
$P_{up} = 2.29558714939263 \dots$	universal parabolic constant
$s = 5.24411510858423 \dots$	arc length of the unit lemniscate
$L = 2.622057554292119 \dots$	lemniscate constant
$L_1 = 1.31102877714605 \dots$	1 <sup>st</sup> lemniscate constant
$L_2 = 0.599070117367796 \dots$	2 <sup>nd</sup> lemniscate constant
$C_U = 0.847213084793979 \dots$	ubiquitous constant
$G_{Ga} = 0.834626841674073 \dots$	Gauss's constant
$W_{We} = 0.47494937998792 \dots$	Weierstrass constant
$C_{R1} = 0.998136044598509 \dots$	Ramanujan's 1 <sup>st</sup> continued fraction constant
$\mathcal{L}_{Li} = 0.110001000000000 \dots$	Liouville's constant
$M = -1.74756459463318 \dots$	Madelung constant
$\mathcal{P} = 0.41468250985111 \dots$	prime constant

$e = 2.71828182845904 \dots$	Euler's number
$x_\infty = 2.29316628741186 \dots$	1 <sup>st</sup> Foias constant
$\gamma = 0.577215664901532 \dots$	Euler-Mascheroni constant
$S = 0.822825249678847 \dots$	Sierpiński's constant
$D_{Do} = 0.739085133215160 \dots$	Dottie number
$L_{LL} = 0.622743419349181 \dots$	Laplace limit
$C_{CFP} = 1.19967864025773 \dots$	Real fixed point of the hyperbolic cotangent
$j_{0,1} = 2.404825557695772 \dots$	1 <sup>st</sup> root of the Bessel function
$\alpha_F = 2.50290787509589 \dots$	alpha Feigenbaum constant
$\delta_F = 4.66920160910299 \dots$	delta Feigenbaum constant
$S = 3.24697960371714 \dots$	silver constant
$P = 1.32471795724474 \dots$	plastic constant
$C_Q = 0.605443657196732 \dots$	QRS constant
$K_{-6} = 1.15655237442151 \dots$	Khinchin mean of order $-6$
$T_0 = 273.150000000000 \dots$ kelvin	freezing point of water
$p_0 = 100,000.000000000 \dots$ pascal	1 bar
$p_1 = 101,325.003754773 \dots$ pascal	standard atmospheric pressure
$E_1 = 6.09110229711386 \dots \times 10^{-24}$ joules	energy associated with $\Delta_{vCs}$
$f_1 = 5.40000000000000 \dots \times 10^{14}$ Hz	frequency of max light conversion
$\lambda_{\text{peak}} = 4.96511423174427 \dots$	peak $\lambda$ of spectral radiance/unit wavelength
$\nu_{\text{peak}} = 2.82143937212207 \dots$	peak emission of spectral flux/unit frequency
$n!$	factorial function
$!n$	derangement function
$\log(x)$	natural logarithm
$\Gamma(x)$	gamma function
$\zeta(x)$	Riemann zeta function

$$\sigma = \frac{X_{\text{pred}} - X_{\text{CODATA}}}{u_{\text{CODATA}}} \quad \Delta_{\text{precision}} = \log_{10} \frac{u_{2018}}{u_{2022}} \quad \Delta_{\text{scaled}} = \frac{X_{2022} - X_{2018}}{u_{2018}}$$

$\sigma = +1.00$  means the prediction is greater than the CODATA value by exactly one of CODATA's error bars;  $\sigma = -1.00$  means it is 1.00 error bar less than the CODATA value, etc. For SI-defined entries we write the match level instead (e.g., "10-digit match").

$\Delta_{\text{precision}} = +1.00$  corresponds to an increase of precision by one significant digit from 2018 to 2022.

$\Delta_{\text{scaled}} = +1.00$  is the signed shift in value from 2018 to 2022, measured in units of the 2018  $1\sigma$  error bars.

$u$  denotes the standard  $1\sigma$  uncertainty from CODATA.

## units

s = second  
m = meter  
C = coulomb  
K = kelvin  
kg = kilogram

## derived units

MHz = mega hertz = 1000000 1/s  
fm = femto meter = 0.000000000000001 m  
eV = electron volt = 1 eV  
MeV = megaelectron volt = 1000000 eV  
GeV = gigaelectron volt = 1000000000 eV

## composite SI units

A = amp  
F = farad  
H = henry  
Hz = hertz  
J = joule  
lm = lumens  
mol = mole  
N = newton  
 $\mathcal{N}$  = noether  
 $\Omega$  = ohm  
Pa = pascal  
S = siemens  
T = tesla  
V = volt  
W = watt  
Wb = weber

the SI unit of electric current  
the SI unit of electrical capacitance  
the SI unit of electrical inductance  
the SI unit of frequency  
the SI unit of energy  
the SI unit of luminous flux  
the SI unit of amount of substance  
the SI unit of force  
the SI unit of momentum  
the SI unit of electrical resistance  
the SI unit of pressure  
the SI unit of electric conductance  
the SI unit of magnetic flux density  
the SI unit of voltage, or electric potential  
the SI unit of power (or radiant flux)  
the SI unit of magnetic flux

A = ampere = C/s  
F = farad =  $s^2 C^2/m^2 kg$   
H = henry =  $m^2 kg/C^2$   
Hz = hertz = 1/s  
J = joule =  $m^2 kg/s^2$   
lm = lumen =  $cd \cdot sr$   
N = newton =  $m kg/s^2$   
 $\mathcal{N}$  = noether =  $m kg/s$   
 $\Omega$  = ohm =  $m^2 kg/s C^2$   
Pa = pascal =  $kg/s^2 m$   
S = siemens =  $s C^2/m^2 kg$   
T = tesla =  $kg/s C$   
V = volt =  $m^2 kg/s^2 C$   
W = watt =  $m^2 kg/s^3$   
Wb = weber =  $m^2 kg/s C$

## hyperfine transition frequency of Cs-133

-5.14637(54)

$$\Delta_{\nu_{\text{Cs}}} = \frac{E_1}{2\pi} \left( \frac{t_p}{l_p^2 m_p} \right) \left( 1 - \sqrt{\zeta(2)} \varkappa_\theta^2 \boxtimes \right)$$

Where  $E_1$  = the energy difference associated with the hyperfine transition frequency,  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\Delta_{\nu_{\text{Cs}}} = 9.19263177042970 \dots \times 10^9 \text{ Hz} \quad \text{prediction}$$

$$\Delta_{\nu_{\text{Cs}}} = 9.192631770 \times 10^9 \text{ Hz} \quad \text{CODATA 2022, 10-digit match}$$

$$\Delta_{\nu_{\text{Cs}}} = 9.192631770 \times 10^9 \text{ Hz} \quad \text{CODATA 2018, 10-digit match}$$

$$\Delta_{\text{precision}} = 0.00$$

$$\Delta_{\text{scaled}} = 0.00$$

$$E_1 = 6.09110229711386 \dots \times 10^{-24} \text{ joules}$$

## electron volt-hertz relationship

-5.1465(21)

$$\text{eV} : \text{Hz} = \frac{\text{eV}}{2\pi} \left( \frac{t_p}{l_p^2 m_p} \right) \left( 1 - \sqrt{\zeta(2)} \varkappa_\theta^2 \boxtimes \right)$$

Where eV = the electron-volt,  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\text{eV} : \text{Hz} = 2.41798924253021 \dots \times 10^{14} \text{ Hz} \quad \text{prediction}$$

$$\text{eV} : \text{Hz} = 2.417989242 \times 10^{14} \text{ Hz} \quad \text{CODATA 2022, 10-digit match}$$

$$\text{eV} : \text{Hz} = 2.417989242 \times 10^{14} \text{ Hz} \quad \text{CODATA 2018, 10-digit match}$$

$$\Delta_{\text{precision}} = 0.00$$

$$\Delta_{\text{scaled}} = 0.00$$

## joule-hertz relationship

-5.1545(76)

$$\text{J} : \text{Hz} = \frac{\text{joule}}{2\pi} \left( \frac{t_p}{l_p^2 m_p} \right) \left( 1 - \sqrt{\zeta(2)} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

J : Hz = 1.50919017971270 ... × 10 <sup>33</sup> Hz	prediction
J : Hz = 1.509190179 × 10 <sup>33</sup> Hz	CODATA 2022, 10-digit match
J : Hz = 1.509190179 × 10 <sup>33</sup> Hz	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = 0.00$
	$\Delta_{\text{scaled}} = 0.00$

## hertz-joule relationship

5.1545(76)

$$\text{Hz} : \text{J} = 2\pi \left( \frac{l_p^2 m_p}{s t_p} \right) \left( 1 + \sqrt{\zeta(2)} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $s$  = the second,  $t_p$  = the Planck time,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

Hz : J = 6.62607014968851 ... × 10 <sup>-34</sup> J	prediction
Hz : J = 6.62607015 × 10 <sup>-34</sup> J	CODATA 2022, 10-digit match
Hz : J = 6.62607015 × 10 <sup>-34</sup> J	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = 0.00$
	$\Delta_{\text{scaled}} = 0.00$

**hertz-electron volt relationship**

5.1465(21)

$$\text{Hz} : \text{eV} = 2\pi \frac{\text{joule}}{\text{eV}} \left( \frac{l_p^2 m_p}{s t_p} \right) \left( 1 + \sqrt{\zeta(2)} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant, eV = the electron-volt,  $l_p$  = the Planck length,  $m_p$  = the Planck mass, s = the second,  $t_p$  = the Planck time,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\text{Hz} : \text{eV} = 4.135667696\mathbf{16115} \dots \times 10^{-15} \text{ eV} \quad \text{prediction}$$

$$\text{Hz} : \text{eV} = 4.135667696 \times 10^{-15} \text{ eV} \quad \text{CODATA 2022, 10-digit match}$$

$$\text{Hz} : \text{eV} = 4.135667696 \times 10^{-15} \text{ eV} \quad \text{CODATA 2018, 10-digit match}$$

$$\Delta_{\text{precision}} = 0.00$$

$$\Delta_{\text{scaled}} = 0.00$$

**natural unit of action in eV s**

5.1507(48)

$$\hbar = \frac{\text{joule}}{\text{eV}} \left( \frac{l_p^2 m_p}{t_p} \right) \left( 1 + \sqrt{\zeta(2)} \varkappa_\theta^2 \boxtimes \right)$$

Where eV = the electron-volt,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\hbar = 6.58211956\mathbf{829518} \dots \times 10^{-16} \text{ eV} \cdot \text{s} \quad \text{prediction}$$

$$\hbar = 6.582119569 \times 10^{-16} \text{ eV} \cdot \text{s} \quad \text{CODATA 2022, 9-digit match}$$

$$\hbar = 6.582119569 \times 10^{-16} \text{ eV} \cdot \text{s} \quad \text{CODATA 2018, 9-digit match}$$

$$\Delta_{\text{precision}} = 0.00$$

$$\Delta_{\text{scaled}} = 0.00$$

**reduced Planck constant**

5.1407(48)

$$\hbar = \left( \frac{l_p^2 m_p}{t_p} \right) \left( 1 + \sqrt{\zeta(2)} \mathfrak{K}_\theta^2 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $\zeta(x)$  = the Riemann zeta function,  $\mathfrak{K}_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\hbar = 1.05457181759658 \dots \times 10^{-34} \text{ J} \cdot \text{s}$	prediction
$\hbar = 1.054571817 \times 10^{-34} \text{ J} \cdot \text{s}$	CODATA 2022, 10-digit match
$\hbar = 1.054571817 \times 10^{-34} \text{ J} \cdot \text{s}$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = 0.00$
	$\Delta_{\text{scaled}} = 0.00$

Also listed as the *atomic unit of action*, and the *natural unit of action*.

**Planck constant**

5.1507(48)

$$h = 2\pi \left( \frac{l_p^2 m_p}{t_p} \right) \left( 1 + \sqrt{\zeta(2)} \mathfrak{K}_\theta^2 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $\zeta(x)$  = the Riemann zeta function,  $\mathfrak{K}_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$h = 6.62607014968851 \dots \times 10^{-34} \text{ J} \cdot \text{s}$	prediction
$h = 6.62607015 \times 10^{-34} \text{ J} \cdot \text{s}$	CODATA 2022, 10-digit match
$h = 6.62607015 \times 10^{-34} \text{ J} \cdot \text{s}$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = 0.00$
	$\Delta_{\text{scaled}} = 0.00$

**quantum of circulation**

5.1470(31)

$$q_c = \pi \left( \frac{l_p^2 m_p}{t_p m_e} \right) \left( 1 + \sqrt{\zeta(2)} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_e$  = the electron mass,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$q_c = 3.63694754800779 \dots \times 10^{-4} \text{ m}^2/\text{s}$	prediction
$q_c = 3.6369475467(11) \times 10^{-4} \text{ m}^2/\text{s}$	CODATA 2022, $\sigma = +1.19$
$q_c = 3.6369475516(11) \times 10^{-4} \text{ m}^2/\text{s}$	CODATA 2018, $\sigma = -3.27$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -4.45$

**quantum of circulation times 2**

5.1470(31)

$$2q_c = 2\pi \left( \frac{l_p^2 m_p}{t_p m_e} \right) \left( 1 + \sqrt{\zeta(2)} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_e$  = the electron mass,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$2q_c = 7.27389509601558 \dots \times 10^{-4} \text{ m}^2/\text{s}$	prediction
$2q_c = 7.2738950934(22) \times 10^{-4} \text{ m}^2/\text{s}$	CODATA 2022, $\sigma = +1.19$
$2q_c = 7.2738951032(22) \times 10^{-4} \text{ m}^2/\text{s}$	CODATA 2018, $\sigma = -3.27$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -4.45$

**Bohr magneton in eV/T**

5.1569(29)

$$\frac{\mu_B}{e} = \frac{1}{2} \left( \frac{l_p^2 C m_p}{t_p m_e} \right) \left( 1 + \sqrt{\zeta(2)} \varkappa_\theta^2 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $C$  = the coulomb,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_e$  = the electron mass,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_B/e = 5.78838180031388 \dots \times 10^{-5} \text{ eV/T}$	prediction
$\mu_B/e = 5.7883817982(18) \times 10^{-5} \text{ eV/T}$	CODATA 2022, $\sigma = +1.17$
$\mu_B/e = 5.7883818060(17) \times 10^{-5} \text{ eV/T}$	CODATA 2018, $\sigma = -3.35$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.59$

**nuclear magneton in eV/T**

5.1471(29)

$$\frac{\mu_N}{e} = \frac{1}{2} \left( \frac{l_p^2 C m_p}{t_p m_+} \right) \left( 1 + \sqrt{\zeta(2)} \varkappa_\theta^2 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $C$  = the coulomb,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_+$  = the proton mass,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_N/e = 3.15245125534845 \dots \times 10^{-8} \text{ eV/T}$	prediction
$\mu_N/e = 3.15245125417(98) \times 10^{-8} \text{ eV/T}$	CODATA 2022, $\sigma = +1.20$
$\mu_N/e = 3.15245125844(96) \times 10^{-8} \text{ eV/T}$	CODATA 2018, $\sigma = -3.22$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.45$

**inverse of conductance quantum**

4.39950(25)

$$G_0^{-1} = \frac{\pi}{\varkappa_1^2} \left( \frac{l_p^2 m_p}{t_p q_p^2} \right) \left( 1 + \sqrt{\zeta(3)} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$G_0^{-1} = 1.29064037306099 \dots \times 10^4 \Omega$	prediction
$G_0^{-1} = 1.290640372 \times 10^4 \Omega$	CODATA 2022, 9-digit match
$G_0^{-1} = 1.290640372 \times 10^4 \Omega$	CODATA 2018, 9-digit match
	$\Delta_{\text{precision}} = 0.00$
	$\Delta_{\text{scaled}} = 0.00$

$\zeta(3)$  = Apéry's constant

**conductance quantum**

-4.39950(25)

$$G_0 = \frac{\varkappa_1^2}{\pi} \left( \frac{t_p q_p^2}{l_p^2 m_p} \right) \left( 1 - \sqrt{\zeta(3)} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$G_0 = 7.74809172928722 \dots \times 10^{-5} \text{ S}$	prediction
$G_0 = 7.748091729 \times 10^{-5} \text{ S}$	CODATA 2022, 10-digit match
$G_0 = 7.748091729 \times 10^{-5} \text{ S}$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = 0.00$
	$\Delta_{\text{scaled}} = 0.00$

**electron volt**

0.3756(31)

$$eV = \varkappa_1 \left( \frac{q_p}{C} \right) \left( 1 + \frac{e^{-\gamma}}{6} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $q_p$  = the Planck charge,  $C$  = the coulomb,  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$eV = 1.60217663422016 \dots \times 10^{-19} \text{ J}$	prediction
$eV = 1.602176634 \times 10^{-19} \text{ J}$	CODATA 2022, 10-digit match
$eV = 1.602176634 \times 10^{-19} \text{ J}$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = 0.00$
	$\Delta_{\text{scaled}} = 0.00$

Also listed as the *electron volt-joule relationship*.

**joule-electron volt relationship**

-0.37485(80)

$$J : eV = \frac{\text{joule}}{\varkappa_1} \left( \frac{C}{q_p} \right) \left( 1 - \frac{e^{-\gamma}}{6} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $q_p$  = the Planck charge,  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$J : eV = 6.24150907360309 \dots \times 10^{18} \text{ eV}$	prediction
$J : eV = 6.241509074 \times 10^{18} \text{ eV}$	CODATA 2022, 10-digit match
$J : eV = 6.241509074 \times 10^{18} \text{ eV}$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = 0.00$
	$\Delta_{\text{scaled}} = 0.00$

**electron charge to mass quotient**

0.3741(30)

$$-\frac{e}{m_e} = -\varkappa_1 \left( \frac{q_p}{m_e} \right) \left( 1 + \frac{e^{-\gamma}}{6} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $q_p$  = the Planck charge,  $m_e$  = the electron mass,  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$-e/m_e = -1.75882000934636 \dots \times 10^{11}$ C/kg	prediction
$-e/m_e = -1.75882000838(55) \times 10^{11}$ C/kg	CODATA 2022, $\sigma = -1.76$
$-e/m_e = -1.75882001076(53) \times 10^{11}$ C/kg	CODATA 2018, $\sigma = -2.67$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.49$

**proton charge to mass quotient**

0.3743(31)

$$\frac{e}{m_+} = \varkappa_1 \left( \frac{q_p}{m_+} \right) \left( 1 + \frac{e^{-\gamma}}{6} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $q_p$  = the Planck charge,  $m_+$  = the proton mass,  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$e/m_+ = 9.57883314831657 \dots \times 10^7$ C/kg	prediction
$e/m_+ = 9.5788331430(29) \times 10^7$ C/kg	CODATA 2022, $\sigma = +1.83$
$e/m_+ = 9.5788331560(29) \times 10^7$ C/kg	CODATA 2018, $\sigma = -2.65$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -4.48$

**Bohr magneton in Hz/T**

0.3841(30)

$$\frac{\mu_B}{h} = \frac{\varkappa_1}{4\pi} \text{tesla}^2 \left( \frac{q_p}{m_e} \right) \left( 1 + \frac{e^{-\gamma}}{6} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $q_p$  = the Planck charge,  $m_e$  = the electron mass,  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_B/h = 1.39962449248203 \dots \times 10^{10} \text{ Hz/T}$	prediction
$\mu_B/h = 1.39962449171(44) \times 10^{10} \text{ Hz/T}$	CODATA 2022, $\sigma = +1.75$
$\mu_B/h = 1.39962449361(42) \times 10^{10} \text{ Hz/T}$	CODATA 2018, $\sigma = -2.69$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.52$

**nuclear magneton in MHz/T**

0.3743(30)

$$\frac{\mu_N}{h} = \frac{\varkappa_1}{4\pi} \frac{1}{\text{MHz}} \left( \frac{q_p}{s m_+} \right) \left( 1 + \frac{e^{-\gamma}}{6} \varkappa_\theta^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant, MHz = a megahertz,  $q_p$  = the Planck charge, s = the second,  $m_+$  = the proton mass,  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_N/h = 7.62259322303543 \dots \text{ MHz/T}$	prediction
$\mu_N/h = 7.6225932188(24) \text{ MHz/T}$	CODATA 2022, $\sigma = +4.68$
$\mu_N/h = 7.6225932291(23) \text{ MHz/T}$	CODATA 2018, $\sigma = -2.64$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.48$

**lattice parameter of silicon**

1343.66(16)

$$a = \frac{4\pi}{32 s} \frac{1}{\varkappa_1^4} \left( \frac{l_p m_p}{m_e} \right) \left( 1 + 35 \left( \frac{4\pi}{8} \right)^5 \varkappa_\theta^2 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$a = 5.43102018763799 \dots \times 10^{-10} \text{ m}$	prediction
$a = 5.431020511(89) \times 10^{-10} \text{ m}$	CODATA 2022, $\sigma = -3.63$
$a = 5.431020511(89) \times 10^{-10} \text{ m}$	CODATA 2018, $\sigma = -3.63$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**lattice spacing of ideal Si (220)**

1343.66(17)

$$d_{220} = \frac{1}{\sqrt{8}} \frac{4\pi}{32 s} \frac{1}{\varkappa_1^4} \left( \frac{l_p m_p}{m_e} \right) \left( 1 + 35 \left( \frac{4\pi}{8} \right)^5 \varkappa_\theta^2 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$d_{220} = 1.92015560171993 \dots \times 10^{-10} \text{ m}$	prediction
$d_{220} = 1.920155716(32) \times 10^{-10} \text{ m}$	CODATA 2022, $\sigma = -3.57$
$d_{220} = 1.920155716(32) \times 10^{-10} \text{ m}$	CODATA 2018, $\sigma = -3.57$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

## Rydberg constant

−8.955076(11)

$$R_\infty = \frac{\varkappa_1^4}{4\pi} \left( \frac{m_e}{l_p m_p} \right) \left( 1 - \frac{1}{8} \left( \frac{6}{V_{fe}} \right)^2 \varkappa_\theta^3 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $m_e$  = the electron mass,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $V_{fe}$  = the figure-eight knot hyperbolic volume,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} R_\infty &= 1.09737315681100 \dots \times 10^7 \text{ 1/m} && \text{prediction} \\ R_\infty &= 1.0973731568157(12) \times 10^7 \text{ 1/m} && \text{CODATA 2022, } \sigma = -3.92 \\ R_\infty &= 1.0973731568160(21) \times 10^7 \text{ 1/m} && \text{CODATA 2018, } \sigma = -2.38 \\ &&& \Delta_{\text{precision}} = +0.24 \\ &&& \Delta_{\text{scaled}} = -0.14 \end{aligned}$$

## hartree-inverse meter relationship

−8.781944(11)

$$E_h \doteq \frac{1}{m} = \frac{\varkappa_1^4}{2\pi} \left( \frac{m_e}{l_p m_p} \right) \left( 1 - \frac{1}{8} \left( \frac{6}{V_{fe}} \right)^2 \varkappa_\theta^3 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $m_e$  = the electron mass,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $V_{fe}$  = the figure-eight knot hyperbolic volume,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} E_h \doteq 1/m &= 2.19474631362199 \dots \times 10^7 \text{ 1/m} && \text{prediction} \\ E_h \doteq 1/m &= 2.1947463136314(24) \times 10^7 \text{ 1/m} && \text{CODATA 2022, } \sigma = -3.92 \\ E_h \doteq 1/m &= 2.1947463136320(43) \times 10^7 \text{ 1/m} && \text{CODATA 2018, } \sigma = -2.33 \\ &&& \Delta_{\text{precision}} = +0.25 \\ &&& \Delta_{\text{scaled}} = -0.14 \end{aligned}$$

## inverse meter-hartree relationship

8.781944(11)

$$\frac{1}{m} : E_h = \frac{2\pi}{\varkappa_1^4} \left( \frac{l_p m_p}{m m_e} \right) \left( 1 + \frac{1}{8} \left( \frac{6}{V_{fe}} \right)^2 \varkappa_\theta^3 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m$  = the meter,  $m_e$  = the electron mass,  $V_{fe}$  = the figure-eight knot hyperbolic volume,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$1/m : E_h = 4.55633525292920 \dots \times 10^{-8} E_h$	prediction
$1/m : E_h = 4.5563352529132(50) \times 10^{-8} E_h$	CODATA 2022, $\sigma = +3.20$
$1/m : E_h = 4.5563352529120(88) \times 10^{-8} E_h$	CODATA 2018, $\sigma = +1.95$
	$\Delta_{\text{precision}} = +0.25$
	$\Delta_{\text{scaled}} = +1.36$

**conventional value of ampere-90**

0.888722(50)

$$A_{90} = \text{ampere} \left( 1 + \frac{1}{S} \left( \frac{4\pi}{35} \right) \varkappa_{\theta}^3 \boxtimes \right)$$

Where  $S$  = the silver constant,  $\pi$  = Archimedes' constant,  $\varkappa_{\theta}$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$A_{90} = 1.00000008888173 \dots \text{ A}$$

prediction

$$A_{90} = 1.00000008887 \text{ A}$$

CODATA 2022, 11.93-digit match

$$A_{90} = 1.00000008887 \text{ A}$$

CODATA 2018, 11.93-digit match

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.00$$

**conventional value of coulomb-90**

0.888750(50)

$$C_{90} = \text{coulomb} \left( 1 + \frac{1}{S} \left( \frac{4\pi}{35} \right) \varkappa_{\theta}^3 \boxtimes \right)$$

Where  $S$  = the silver constant,  $\pi$  = Archimedes' constant,  $\varkappa_{\theta}$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$C_{90} = 1.00000008888173 \dots \text{ C}$$

prediction

$$C_{90} = 1.00000008887 \text{ C}$$

CODATA 2022, 11.93-digit match

$$C_{90} = 1.00000008887 \text{ C}$$

CODATA 2018, 11.93-digit match

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.00$$

**Bohr magneton in inverse meter per tesla**

3.9855(30)

$$\frac{\mu_B}{hc} = \frac{\varkappa_1}{4\pi} \left( \frac{t_p q_p}{l_p m_e} \right) \left( 1 + \mathcal{S} \left( \frac{K}{6} \right) \varkappa_\theta^3 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $\mathcal{S}$  = the silver constant,  $K$  = Catalan's constant,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_B/hc = 4.66864477302116 \dots \times 10^1 \text{ 1/m} \cdot \text{T}$	prediction
$\mu_B/hc = 4.6686447719(15) \times 10^1 \text{ 1/m} \cdot \text{T}$	CODATA 2022, $\sigma = +0.75$
$\mu_B/hc = 4.6686447783(14) \times 10^1 \text{ 1/m} \cdot \text{T}$	CODATA 2018, $\sigma = -3.77$
	$\Delta_{\text{precision}} = -0.07$
	$\Delta_{\text{scaled}} = -4.57$

**nuclear magneton in inverse meter per tesla**

3.9855(30)

$$\frac{\mu_N}{hc} = \frac{\varkappa_1}{4\pi} \left( \frac{t_p q_p}{l_p m_+} \right) \left( 1 + \mathcal{S} \left( \frac{K}{6} \right) \varkappa_\theta^3 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $l_p$  = the Planck length,  $m_+$  = the proton mass,  $\mathcal{S}$  = the silver constant,  $K$  = Catalan's constant,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_N/hc = 2.54262341068940 \dots \times 10^{-2} \text{ 1/m} \cdot \text{T}$	prediction
$\mu_N/hc = 2.54262341009(79) \times 10^{-2} \text{ 1/m} \cdot \text{T}$	CODATA 2022, $\sigma = +0.76$
$\mu_N/hc = 2.54262341353(78) \times 10^{-2} \text{ 1/m} \cdot \text{T}$	CODATA 2018, $\sigma = -3.64$
	$\Delta_{\text{precision}} = -0.13$
	$\Delta_{\text{scaled}} = -4.41$

**Coulomb's constant**

0.7816(15)

$$\kappa = \left( \frac{l_p^3 m_p}{t_p^2 q_p^2} \right) \left( 1 + \left( \frac{C_d}{6} \right) \mathfrak{K}_\theta^4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $C_d$  = the domino tiling constant,  $\mathfrak{K}_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\kappa = 8.98755179302985 \dots \times 10^9 \text{ m/F}$	prediction
$\kappa = 8.9875517923(14) \times 10^9 \text{ m/F}$	Wikipedia 2022, $\sigma = +0.52$
$\kappa = 8.9875517923(14) \times 10^9 \text{ m/F}$	CODATA 2018, $\sigma = +0.52$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**atomic unit of permittivity**

-0.7815(15)

$$A_{\text{perm}} = \left( \frac{t_p^2 q_p^2}{l_p^3 m_p} \right) \left( 1 - \left( \frac{C_d}{6} \right) \mathfrak{K}_\theta^4 \boxtimes \right)$$

Where  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $C_d$  = the domino tiling constant,  $\mathfrak{K}_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{perm}} = 1.11265005535270 \dots \times 10^{-10} \text{ F/m}$	prediction
$A_{\text{perm}} = 1.11265005620(17) \times 10^{-10} \text{ F/m}$	CODATA 2022, $\sigma = -4.98$
$A_{\text{perm}} = 1.11265005545(17) \times 10^{-10} \text{ F/m}$	CODATA 2018, $\sigma = -4.78$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +4.41$

**vacuum electric permittivity**

-0.7815(14)

$$\varepsilon_0 = \frac{1}{4\pi} \left( \frac{t_p^2 q_p^2}{l_p^3 m_p} \right) \left( 1 - \left( \frac{C_d}{6} \right) \varkappa_\theta^4 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $C_d$  = the domino tiling constant,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\varepsilon_0 = 8.85418781204269 \dots \times 10^{-12}$ F/m	prediction
$\varepsilon_0 = 8.8541878188(14) \times 10^{-12}$ F/m	CODATA 2022, $\sigma = -4.83$
$\varepsilon_0 = 8.8541878128(13) \times 10^{-12}$ F/m	CODATA 2018, $\sigma = -0.58$
	$\Delta_{\text{precision}} = -0.03$
	$\Delta_{\text{scaled}} = +4.62$

Also called *vacuum permittivity*, or the *electric constant*.

**atomic unit of time**

12.390554(10)

$$A_{\text{time}} = \frac{1}{\kappa_1^4} \left( \frac{t_p m_p}{m_e} \right) \left( 1 + \sqrt{5} \left( \frac{C_d}{C_U} \right) \kappa_\theta^4 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $t_p$  = the Planck time,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $C_d$  = the domino tiling constant,  $C_U$  = the ubiquitous constant,  $\kappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} A_{\text{time}} &= 2.41888432657500 \dots \times 10^{-17} \text{ s} && \text{prediction} \\ A_{\text{time}} &= 2.4188843265864(26) \times 10^{-17} \text{ s} && \text{CODATA 2022, } \sigma = -4.39 \\ A_{\text{time}} &= 2.4188843265857(47) \times 10^{-17} \text{ s} && \text{CODATA 2018, } \sigma = -2.28 \\ &&& \Delta_{\text{precision}} = +0.26 \\ &&& \Delta_{\text{scaled}} = +0.15 \end{aligned}$$

**hertz-hartree relationship**

12.390554(10)

$$\text{Hz} : E_h = \text{watt} \frac{2\pi}{\kappa_1^4} \left( \frac{t_p m_p}{m_e} \right) \left( 1 + \sqrt{5} \left( \frac{C_d}{C_U} \right) \kappa_\theta^4 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $t_p$  = the Planck time,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $C_d$  = the domino tiling constant,  $C_U$  = the ubiquitous constant,  $\kappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \text{Hz} : E_h &= 1.51982984605030 \dots \times 10^{-16} E_h && \text{prediction} \\ \text{Hz} : E_h &= 1.5198298460574(17) \times 10^{-16} E_h && \text{CODATA 2022, } \sigma = -4.18 \\ \text{Hz} : E_h &= 1.5198298460570(29) \times 10^{-16} E_h && \text{CODATA 2018, } \sigma = -2.31 \\ &&& \Delta_{\text{precision}} = +0.23 \\ &&& \Delta_{\text{scaled}} = +1.14 \end{aligned}$$

## hartree-hertz relationship

-12.390554(10)

$$E_h \dot{:} \text{Hz} = \frac{\varkappa_1^4}{2\pi} \left( \frac{m_e}{t_p m_p} \right) \left( 1 - \sqrt{5} \left( \frac{C_d}{C_U} \right) \varkappa_\theta^4 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $m_p$  = the Planck mass,  $C_d$  = the domino tiling constant,  $C_U$  = the ubiquitous constant,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} E_h \dot{:} \text{Hz} &= 6.57968392052072 \dots \times 10^{15} \text{ Hz} && \text{prediction} \\ E_h \dot{:} \text{Hz} &= 6.5796839204999(72) \times 10^{15} \text{ Hz} && \text{CODATA 2022, } \sigma = +2.89 \\ E_h \dot{:} \text{Hz} &= 6.579683920502(13) \times 10^{15} \text{ Hz} && \text{CODATA 2018, } \sigma = +1.44 \\ &&& \Delta_{\text{precision}} = +0.74 \\ &&& \Delta_{\text{scaled}} = -0.23 \end{aligned}$$

## Rydberg constant times c in Hz

-12.390554(10)

$$R_\infty \dot{c} = \frac{\varkappa_1^4}{4\pi} \left( \frac{m_e}{t_p m_p} \right) \left( 1 - \sqrt{5} \left( \frac{C_d}{C_U} \right) \varkappa_\theta^4 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $m_p$  = the Planck mass,  $C_d$  = the domino tiling constant,  $C_U$  = the ubiquitous constant,  $\varkappa_\theta$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} R_\infty \dot{c} &= 3.28984196026036 \dots \times 10^{15} \text{ Hz} && \text{prediction} \\ R_\infty \dot{c} &= 3.2898419602500(36) \times 10^{15} \text{ Hz} && \text{CODATA 2022, } \sigma = +2.88 \\ R_\infty \dot{c} &= 3.2898419602508(64) \times 10^{15} \text{ Hz} && \text{CODATA 2018, } \sigma = +1.49 \\ &&& \Delta_{\text{precision}} = +0.25 \\ &&& \Delta_{\text{scaled}} = -0.13 \end{aligned}$$

**neutron-proton magnetic moment ratio**

760.3(23)

$$\frac{\mu_n}{\mu_+} = -\frac{S}{C_{\text{CFP}}}\left(\frac{m_+}{m_n}\right)\left(1 + !5\left(\frac{2\pi}{8K}\right)\varkappa_r^2 \boxtimes\right)$$

Where  $S$  = Sierpiński's constant,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $m_+$  = the proton mass,  $m_n$  = the neutron mass,  $!n$  = the derangement function,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_n/\mu_+ = -6.84979091085629 \dots \times 10^{-1}$	prediction
$\mu_n/\mu_+ = -6.8497935(16) \times 10^{-1}$	CODATA 2018, $\sigma = +1.63$
$\mu_n/\mu_+ = -6.8497934(16) \times 10^{-1}$	CODATA 2022, $\sigma = -1.56$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.06$

**proton-neutron magnetic moment ratio**

-760.3(23)

$$\frac{\mu_+}{\mu_n} = -\frac{C_{\text{CFP}}}{S}\left(\frac{m_n}{m_+}\right)\left(1 + !5\left(\frac{2\pi}{8K}\right)\varkappa_r^2 \boxtimes\right)^{-1}$$

Where  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $S$  = Sierpiński's constant,  $m_n$  = the neutron mass,  $m_+$  = the proton mass,  $!n$  = the derangement function,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_+/\mu_n = -1.45989857648807 \dots$	prediction
$\mu_+/\mu_n = -1.45989802(34)$	CODATA 2018, $\sigma = -1.64$
$\mu_+/\mu_n = -1.45989805(34)$	CODATA 2018, $\sigma = -1.55$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.09$

**electron-neutron magnetic moment ratio**

213.9(24)

$$\frac{\mu_e}{\mu_n} = \frac{C_{\text{CFP}}}{P_{up}} \left( \frac{m_n}{m_e} \right) \left( 1 + 18 \left( \frac{4K}{2\pi} \right) \varkappa_r^2 \boxtimes \right)$$

Where  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $P_{up}$  = the universal parabolic constant,  $m_n$  = the neutron mass,  $m_e$  = the electron mass,  $K$  = Catalan's constant,  $\pi$  = Archimedes' constant,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_e/\mu_n = 9.60920149475115 \dots \times 10^2$	prediction
$\mu_e/\mu_n = 9.6092048(23) \times 10^2$	CODATA 2022, $\sigma = -1.44$
$\mu_e/\mu_n = 9.6092050(23) \times 10^2$	CODATA 2018, $\sigma = -1.53$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.09$

**neutron-electron magnetic moment ratio**

-213.9(24)

$$\frac{\mu_n}{\mu_e} = \frac{P_{up}}{C_{\text{CFP}}} \left( \frac{m_e}{m_n} \right) \left( 1 + 18 \left( \frac{4K}{2\pi} \right) \varkappa_r^2 \boxtimes \right)^{-1}$$

Where  $P_{up}$  = the universal parabolic constant,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $m_e$  = the electron mass,  $m_n$  = the neutron mass,  $K$  = Catalan's constant,  $\pi$  = Archimedes' constant,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_n/\mu_e = 1.04066919665097 \dots \times 10^{-3}$	prediction
$\mu_n/\mu_e = 1.04066884(24) \times 10^{-3}$	CODATA 2022, $\sigma = +1.49$
$\mu_n/\mu_e = 1.04066882(25) \times 10^{-3}$	CODATA 2018, $\sigma = +1.50$
	$\Delta_{\text{precision}} = +0.02$
	$\Delta_{\text{scaled}} = +0.04$

**atomic unit of magnetic dipole moment**

5.5204(31)

$$A_{\text{mdm}} = \varkappa_1 \left( \frac{l_p^2 q_p m_p}{t_p m_e} \right) \left( 1 + \omega_2 \left( \frac{2\pi}{35} \right) \varkappa_r^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_e$  = the electron mass,  $\omega_2$  = the omega\_2 constant,  $\pi$  = Archimedes' constant,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{mdm}} = 1.85480201153128 \dots \times 10^{-23} \text{ J/T}$	prediction
$A_{\text{mdm}} = 1.85480201315(58) \times 10^{-23} \text{ J/T}$	CODATA 2022, $\sigma = -2.79$
$A_{\text{mdm}} = 1.85480201566(56) \times 10^{-23} \text{ J/T}$	CODATA 2018, $\sigma = -7.37$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.48$

**Bohr magneton**

5.5208(31)

$$\mu_B = \frac{\varkappa_1}{2} \left( \frac{l_p^2 q_p m_p}{t_p m_e} \right) \left( 1 + \omega_2 \left( \frac{2\pi}{35} \right) \varkappa_r^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_e$  = the electron mass,  $\omega_2$  = the omega\_2 constant,  $\pi$  = Archimedes' constant,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_B = 9.27401005765638 \dots \times 10^{-24} \text{ J/T}$	prediction
$\mu_B = 9.2740100657(29) \times 10^{-24} \text{ J/T}$	CODATA 2022, $\sigma = -2.77$
$\mu_B = 9.2740100783(28) \times 10^{-24} \text{ J/T}$	CODATA 2018, $\sigma = -7.37$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.50$

**nuclear magneton**

5.5208(31)

$$\mu_N = \frac{\varkappa_1}{2} \left( \frac{l_p^2 q_p m_p}{t_p m_+} \right) \left( 1 + \omega_2 \left( \frac{2\pi}{35} \right) \varkappa_r^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_+$  = the proton mass,  $\omega_2$  = the omega\_2 constant,  $\pi$  = Archimedes' constant,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_N = 5.05078373489248 \dots \times 10^{-27}$ J/T	prediction
$\mu_N = 5.0507837393(16) \times 10^{-27}$ J/T	CODATA 2022, $\sigma = -2.75$
$\mu_N = 5.0507837461(15) \times 10^{-27}$ J/T	CODATA 2018, $\sigma = -7.47$
	$\Delta_{\text{precision}} = -0.03$
	$\Delta_{\text{scaled}} = -4.53$

**triton magnetic moment**

-10.716(20)

$$\mu_{\text{tri}} = \varkappa_1^2 \varkappa_2^2 (D_{\text{Do}} + \pi) \left( \frac{l_p^2 q_p m_p}{t_p m_{\text{tri}}} \right) \left( 1 - \omega_2 \left( \frac{2\pi}{18} \right) \varkappa_r^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $D_{\text{Do}}$  = the Dottie number,  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_{\text{tri}}$  = the triton mass,  $\omega_2$  = the omega\_2 constant,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_{\text{tri}} = 1.50460951611437 \dots \times 10^{-26}$ J/T	prediction
$\mu_{\text{tri}} = 1.5046095178(30) \times 10^{-26}$ J/T	CODATA 2022, $\sigma = -0.56$
$\mu_{\text{tri}} = 1.5046095202(30) \times 10^{-26}$ J/T	CODATA 2018, $\sigma = -1.36$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.80$

**triton to proton magnetic moment ratio**

308.242(19)

$$\frac{\mu_{\text{tri}}}{\mu_+} = S ( D_{\text{Do}} + \pi ) \left( \frac{m_+}{m_{\text{tri}}} \right) \left( 1 + \frac{4}{L_{LL}} \left( \frac{32}{4\pi} \right) \varkappa_r^2 \boxtimes \right)$$

Where  $S$  = Sierpiński's constant,  $D_{\text{Do}}$  = the Dottie number,  $\pi$  = Archimedes' constant,  $m_+$  = the proton mass,  $m_{\text{tri}}$  = the triton mass,  $L_{LL}$  = the Laplace limit,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\mu_{\text{tri}}/\mu_+ = 1.06663991754137 \dots$$

prediction

$$\mu_{\text{tri}}/\mu_+ = 1.0666399189(21)$$

CODATA 2022,  $\sigma = -0.65$ 

$$\mu_{\text{tri}}/\mu_+ = 1.0666399191(21)$$

CODATA 2018,  $\sigma = -0.74$ 

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = -0.10$$

**triton magnetic moment to nuclear magneton ratio**

-16.237(20)

$$\frac{\mu_{\text{tri}}}{\mu_N} = \varkappa_1 \varkappa_2^2 ( 2D_{\text{Do}} + 2\pi ) \left( \frac{m_+}{m_{\text{tri}}} \right) \left( 1 - \frac{18}{8} \left( \frac{4\pi}{35} \right) \varkappa_r^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $D_{\text{Do}}$  = the Dottie number,  $\pi$  = Archimedes' constant,  $m_+$  = the proton mass,  $m_{\text{tri}}$  = the triton mass,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\mu_{\text{tri}}/\mu_N = 2.97896247563036 \dots$$

prediction

$$\mu_{\text{tri}}/\mu_N = 2.9789624650(59)$$

CODATA 2022,  $\sigma = +1.80$ 

$$\mu_{\text{tri}}/\mu_N = 2.9789624656(59)$$

CODATA 2018,  $\sigma = +1.70$ 

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = -0.10$$

**triton magnetic moment to Bohr magneton ratio**

-16.237(19)

$$\frac{\mu_{\text{tri}}}{\mu_B} = \mathfrak{K}_1 \mathfrak{K}_2^2 (2D_{\text{Do}} + 2\pi) \left( \frac{m_e}{m_{\text{tri}}} \right) \left( 1 - \frac{18}{8} \left( \frac{4\pi}{35} \right) \mathfrak{K}_r^2 \boxtimes \right)$$

Where  $\mathfrak{K}_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $D_{\text{Do}}$  = the Dottie number,  $\pi$  = Archimedes' constant,  $m_e$  = the electron mass,  $m_{\text{tri}}$  = the triton mass,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_{\text{tri}}/\mu_B = 1.62239367061578 \dots \times 10^{-3}$	prediction
$\mu_{\text{tri}}/\mu_B = 1.6223936648(32) \times 10^{-3}$	CODATA 2022, $\sigma = +1.82$
$\mu_{\text{tri}}/\mu_B = 1.6223936651(32) \times 10^{-3}$	CODATA 2018, $\sigma = +1.72$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.09$

**triton g factor**

-16.236(20)

$$g_{\text{tri}} = \mathfrak{K}_1 \mathfrak{K}_2^2 (4D_{\text{Do}} + 4\pi) \left( \frac{m_+}{m_{\text{tri}}} \right) \left( 1 - \frac{18}{8} \left( \frac{4\pi}{35} \right) \mathfrak{K}_r^2 \boxtimes \right)$$

Where  $\mathfrak{K}_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $D_{\text{Do}}$  = the Dottie number,  $\pi$  = Archimedes' constant,  $m_+$  = the proton mass,  $m_{\text{tri}}$  = the triton mass,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$g_{\text{tri}} = 5.95792495126071 \dots$	prediction
$g_{\text{tri}} = 5.957924930(12)$	CODATA 2022, $\sigma = +1.77$
$g_{\text{tri}} = 5.957924931(12)$	CODATA 2018, $\sigma = -1.69$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.08$

**electron-muon magnetic moment ratio**

-62.35(22)

$$\frac{\mu_e}{\mu_\mu} = \left( \frac{m_\mu}{m_e} \right) \left( 1 - \left( \frac{4\pi}{18} \right) \mathfrak{K}_r^3 \boxtimes \right)$$

Where  $m_\mu$  = the muon mass,  $m_e$  = the electron mass,  $\pi$  = Archimedes' constant,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_e/\mu_\mu = 2.06766985041323 \dots \times 10^2$	prediction
$\mu_e/\mu_\mu = 2.067669881(46) \times 10^2$	CODATA 2022, $\sigma = +0.66$
$\mu_e/\mu_\mu = 2.067669883(46) \times 10^2$	CODATA 2018, $\sigma = -0.71$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.04$

**Newtonian constant of gravitation**

-60.(103)

$$G = \left( \frac{l_p^3}{t_p^2 m_p} \right) \left( 1 - \left( \frac{4\pi}{18} \right) \mathfrak{K}_r^3 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $t_p$  = the Planck time,  $m_p$  = the Planck mass,  $\pi$  = Archimedes' constant,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$G = 6.67430315504533 \dots \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg}$	prediction
$G = 6.67430(15) \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg}$	CODATA 2022, $\sigma = +0.02$
$G = 6.67430(15) \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg}$	CODATA 2018, $\sigma = +0.02$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

Also called the *gravitational constant*.

## Newtonian constant of gravitation over $\hbar c$

-70 ±223

$$\frac{G}{\hbar c} = \text{GeV}^2 \left( \frac{t_p^4}{l_p^4 m_p^2} \right) \left( 1 - \left( \frac{4\pi}{18} \right) \kappa_r^3 \boxtimes \right)$$

Where GeV = the gigaelectron volt,  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $\pi$  = Archimedes' constant,  $\kappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$G/\hbar c = 6.70882525604658 \dots \times 10^{-39} \text{ c}^4/\text{GeV}^2$	prediction
$G/\hbar c = 6.70883(15) \times 10^{-39} \text{ c}^4/\text{GeV}^2$	CODATA 2022, $\sigma = -0.03$
$G/\hbar c = 6.70883(15) \times 10^{-39} \text{ c}^4/\text{GeV}^2$	CODATA 2018, $\sigma = -0.03$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

The Planck energy is equal to  $E_p = \frac{l_p^2 m_p}{t_p^2}$ .

**neutron mass**

-74.9526(39)

$$m_n = \left(\frac{2\pi}{2s}\right)^2 \frac{\text{GeV}}{\varphi^{-2}} \left(\frac{t_p^2}{l_p^2}\right) \left(1 - \left(\frac{2s}{4\pi}\right) \mathfrak{K}_r^3 \boxtimes\right)$$

Where  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate, GeV = the gigaelectron volt,  $\varphi$  = the golden ratio,  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_n = 1.67492749887876 \dots \times 10^{-27} \text{ kg}$	prediction
$m_n = 1.67492750056(85) \times 10^{-27} \text{ kg}$	CODATA 2022, $\sigma = -1.98$
$m_n = 1.67492749804(95) \times 10^{-27} \text{ kg}$	CODATA 2018, $\sigma = +0.88$
	$\Delta_{\text{precision}} = +0.05$
	$\Delta_{\text{scaled}} = +2.65$

**proton mass**

-179.826407(84)

$$m_+ = \left(\frac{2\pi}{s}\right)^2 \frac{\text{GeV}}{\omega_2} \left(\frac{t_p^2}{l_p^2}\right) \left(1 - \frac{\sqrt{2}}{e^{\gamma/3}} \left(\frac{18}{2s}\right) \mathfrak{K}_r^3 \boxtimes\right)$$

Where  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate, GeV = the gigaelectron volt,  $\omega_2$  = the omega\_2 constant,  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_+ = 1.67262192525165 \dots \times 10^{-27} \text{ kg}$	prediction
$m_+ = 1.67262192595(52) \times 10^{-27} \text{ kg}$	CODATA 2022, $\sigma = -1.34$
$m_+ = 1.67262192369(51) \times 10^{-27} \text{ kg}$	CODATA 2018, $\sigma = +3.06$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = +4.35$

**deuteron mass**

130.771469(75)

$$m_{\text{de}} = \frac{18}{6 C_{R1}} \sqrt{\frac{2\pi}{14}} (A_{\text{mass}}) \left( 1 + (\pi - x_{\infty}) \left( \frac{18}{2s} \right) \mathfrak{K}_r^3 \boxtimes \right)$$

Where  $C_{R1}$  = Ramanujan's 1<sup>st</sup> continued fraction constant,  $\pi$  = Archimedes' constant,  $A_{\text{mass}}$  = the atomic mass constant,  $x_{\infty}$  = the 1<sup>st</sup> Foias constant,  $s$  = the arc length of the unit lemniscate,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_{\text{de}} = 3.34358377536356 \dots \times 10^{-27}$ kg	prediction
$m_{\text{de}} = 3.3435837768(10) \times 10^{-27}$ kg	CODATA 2022, $\sigma = -1.44$
$m_{\text{de}} = 3.3435837724(10) \times 10^{-27}$ kg	CODATA 2018, $\sigma = +2.96$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +4.40$

**muon mass**

-1869.61(23)

$$m_{\mu} = \frac{2\pi}{35} \sqrt{\frac{4\pi}{6s}} (A_{\text{mass}}) \left( 1 - 18 K_{-6} \mathfrak{K}_r^3 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $A_{\text{mass}}$  = the atomic mass constant,  $K_{-6}$  = the Khinchin mean of order  $-6$ ,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_{\mu} = 1.88353161502363 \dots \times 10^{-28}$ kg	prediction
$m_{\mu} = 1.883531627(42) \times 10^{-28}$ kg	CODATA 2022, $\sigma = -0.29$
$m_{\mu} = 1.883531627(42) \times 10^{-28}$ kg	CODATA 2018, $\sigma = -0.29$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**deuteron-electron magnetic moment ratio**

-1851.087(25)

$$\frac{\mu_{de}}{\mu_e} = -P_{up} \sqrt{\frac{2s}{6\pi} \left( \frac{m_e}{m_{de}} \right) \left( 1 - 18 \sqrt{L_1} \varkappa_r^3 \boxtimes \right)}$$

Where  $P_{up}$  = the universal parabolic constant,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $m_e$  = the electron mass,  $m_{de}$  = the deuteron mass,  $L_1$  = the 1<sup>st</sup> lemniscate constant,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_{de}/\mu_e = -4.66434558972163 \dots \times 10^{-4}$	prediction
$\mu_{de}/\mu_e = -4.664345550(12) \times 10^{-4}$	CODATA 2022, $\sigma = -3.31$
$\mu_{de}/\mu_e = -4.664345551(12) \times 10^{-4}$	CODATA 2018, $\sigma = -3.23$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.08$

**electron-deuteron magnetic moment ratio**

1851.431(26)

$$\frac{\mu_e}{\mu_{de}} = -\frac{1}{P_{up}} \sqrt{\frac{6\pi}{2s} \left( \frac{m_{de}}{m_e} \right) \left( 1 - 18 \sqrt{L_1} \varkappa_r^3 \boxtimes \right)^{-1}}$$

Where  $P_{up}$  = the universal parabolic constant,  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $m_{de}$  = the deuteron mass,  $m_e$  = the electron mass,  $L_1$  = the 1<sup>st</sup> lemniscate constant,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_e/\mu_{de} = -2.14392347386009 \dots \times 10^3$	prediction
$\mu_e/\mu_{de} = -2.1439234921(56) \times 10^3$	CODATA 2022, $\sigma = +3.26$
$\mu_e/\mu_{de} = -2.1439234915(56) \times 10^3$	CODATA 2018, $\sigma = 3.15$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.11$

**deuteron-proton magnetic moment ratio**

-876.881(26)

$$\frac{\mu_{de}}{\mu_+} = S \sqrt{\frac{2s}{6\pi} \left( \frac{m_+}{m_{de}} \right)} \left( 1 - \frac{!5}{\varphi} \left( \frac{4\pi}{35} \right) \mathfrak{K}_r^3 \boxtimes \right)$$

Where  $S$  = Sierpiński's constant,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $m_+$  = the proton mass,  $m_{de}$  = the deuteron mass,  $!n$  = the derangement function,  $\varphi$  = the golden ratio,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_{de}/\mu_+ = 3.07012209390825 \dots \times 10^{-1}$	prediction
$\mu_{de}/\mu_+ = 3.0701220930(79) \times 10^{-1}$	CODATA 2022, $\sigma = +0.11$
$\mu_{de}/\mu_+ = 3.0701220939(79) \times 10^{-1}$	CODATA 2018, $\sigma = +0.00$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.11$

**deuteron-neutron magnetic moment ratio**

-1637.0(25)

$$\frac{\mu_{de}}{\mu_n} = -C_{\text{CFP}} \sqrt{\frac{2s}{6\pi} \left( \frac{m_n}{m_{de}} \right)} \left( 1 - 35 \text{Im}(\omega_1) \left( \frac{4\pi}{32} \right) \mathfrak{K}_r^3 \boxtimes \right)$$

Where  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $m_n$  = the neutron mass,  $m_{de}$  = the deuteron mass,  $\omega_1$  = the omega\_1 constant,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_{de}/\mu_n = -4.48206589662326 \dots \times 10^{-1}$	prediction
$\mu_{de}/\mu_n = -4.4820652(11) \times 10^{-1}$	CODATA 2022, $\sigma = -0.63$
$\mu_{de}/\mu_n = -4.4820653(11) \times 10^{-1}$	CODATA 2018, $\sigma = -0.54$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.09$

**deuteron magnetic moment**

-1195.796(25)

$$\mu_{de} = \kappa_1^2 \kappa_2^2 \sqrt{\frac{2s}{6\pi} \left( \frac{l_p^2 q_p m_p}{t_p m_{de}} \right)} \left( 1 - \frac{8}{\omega_2} \left( \frac{32}{4\pi} \right) \kappa_r^3 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = 2<sup>nd</sup> hyperbolic partition constant,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_{de}$  = the deuteron mass,  $\omega_2$  = the omega\_2 constant,  $\kappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_{de} = 4.33073505993240 \times 10^{-27} \text{ J/T}$	prediction
$\mu_{de} = 4.330735087(11) \times 10^{-27} \text{ J/T}$	CODATA 2022, $\sigma = -2.46$
$\mu_{de} = 4.330735094(11) \times 10^{-27} \text{ J/T}$	CODATA 2018, $\sigma = -3.10$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.63$

## shielded helion magnetic moment to Bohr magneton ratio

347.5705(81)

$$\frac{\mu_{\text{he}}'}{\mu_B} = -\frac{1}{2} \sqrt{\frac{1}{4\pi} \frac{1}{4s} \left( \frac{m_\mu}{m_{\text{he}}} \right) \left( 1 + \frac{1}{\sqrt{7}} \left( \frac{2\pi}{3K} \right) \mathfrak{K}_r^4 \boxtimes \right)}$$

Where  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $m_\mu$  = the muon mass,  $m_{\text{he}}$  = the helion mass,  $K$  = Catalan's constant,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \mu_{\text{he}}'/\mu_B &= -1.15867149694878 \dots \times 10^{-3} && \text{prediction} \\ \mu_{\text{he}}'/\mu_B &= -1.15867149457(94) \times 10^{-3} && \text{CODATA 2022, } \sigma = -2.53 \\ \mu_{\text{he}}'/\mu_B &= -1.158671471(14) \times 10^{-3} && \text{CODATA 2018, } \sigma = -1.85 \\ &&& \Delta_{\text{precision}} = +2.17 \\ &&& \Delta_{\text{scaled}} = +1.64 \end{aligned}$$

## shielded helion magnetic moment to nuclear magneton ratio

347.5705(81)

$$\frac{\mu_{\text{he}}'}{\mu_N} = -\frac{1}{2} \sqrt{\frac{1}{4\pi} \frac{1}{4s} \left( \frac{m_+ m_\mu}{m_e m_{\text{he}}} \right) \left( 1 + \frac{1}{\sqrt{7}} \left( \frac{2\pi}{3K} \right) \mathfrak{K}_r^4 \boxtimes \right)}$$

Where  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $m_+$  = the proton mass,  $m_\mu$  = the muon mass,  $m_e$  = the electron mass,  $m_{\text{he}}$  = the helion mass,  $K$  = Catalan's constant,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \mu_{\text{he}}'/\mu_N &= -2.12749776673056 \dots && \text{prediction} \\ \mu_{\text{he}}'/\mu_N &= -2.1274977624(17) && \text{CODATA 2022, } \sigma = -2.55 \\ \mu_{\text{he}}'/\mu_N &= -2.127497719(25) && \text{CODATA 2018, } \sigma = -2.07 \\ &&& \Delta_{\text{precision}} = +1.17 \\ &&& \Delta_{\text{scaled}} = +1.72 \end{aligned}$$

# helion g factor

947.298(19)

$$g_{\text{he}} = -\sqrt{\frac{1}{4\pi} \frac{1}{4s} \left( \frac{m_+ m_\mu}{m_e m_{\text{he}}} \right) \left( 1 + 18 \left( \frac{2K}{14} \right) \mathfrak{K}_r^4 \boxtimes \right)}$$

Where  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $m_+$  = the proton mass,  $m_\mu$  = the muon mass,  $m_e$  = the electron mass,  $m_{\text{he}}$  = the helion mass,  $K$  = Catalan's constant,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$g_{\text{he}} = -4.25525070422298 \dots$$

$$g_{\text{he}} = -4.2552506995(34)$$

$$g_{\text{he}} = -4.255250615(50)$$

prediction

CODATA 2022,  $\sigma = -1.39$

CODATA 2018,  $\sigma = -1.78$

$\Delta_{\text{precision}} = +0.17$

$\Delta_{\text{scaled}} = +1.16$

**helion magnetic moment to nuclear magneton ratio** 947.2981(82)

$$\frac{\mu_{\text{he}}}{\mu_N} = -\frac{1}{2} \sqrt{\frac{1}{4\pi} \frac{1}{4s} \left( \frac{m_+ m_\mu}{m_e m_{\text{he}}} \right)} \left( 1 + 18 \left( \frac{2K}{14} \right) \mathfrak{K}_r^4 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $m_+$  = the proton mass,  $m_\mu$  = the muon mass,  $m_e$  = the electron mass,  $m_{\text{he}}$  = the helion mass,  $K$  = Catalan's constant,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_{\text{he}}/\mu_N = -2.12762535211149$	prediction
$\mu_{\text{he}}/\mu_N = -2.1276253498(17)$	CODATA 2018, $\sigma = -1.36$
$\mu_{\text{he}}/\mu_N = -2.127625307(25)$	CODATA 2018, $\sigma = -1.80$
	$\Delta_{\text{precision}} = +1.17$
	$\Delta_{\text{scaled}} = +1.16$

**helion magnetic moment to Bohr magneton ratio** 947.2981(82)

$$\frac{\mu_{\text{he}}}{\mu_B} = -\frac{1}{2} \sqrt{\frac{1}{4\pi} \frac{1}{4s} \left( \frac{m_\mu}{m_{\text{he}}} \right)} \left( 1 + 18 \left( \frac{2K}{14} \right) \mathfrak{K}_r^4 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $m_\mu$  = the muon mass,  $m_{\text{he}}$  = the helion mass,  $K$  = Catalan's constant,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_{\text{he}}/\mu_B = -1.15874098211893 \dots \times 10^{-3}$	prediction
$\mu_{\text{he}}/\mu_B = -1.15874098083(94) \times 10^{-3}$	CODATA 2022, $\sigma = -1.37$
$\mu_{\text{he}}/\mu_B = -1.158740958(14) \times 10^{-3}$	CODATA 2018, $\sigma = -1.72$
	$\Delta_{\text{precision}} = +1.17$
	$\Delta_{\text{scaled}} = +1.57$

**vacuum magnetic permeability**

8.0057(15)

$$\mu_0 = 4\pi \left( \frac{l_p m_p}{q_p^2} \right) \left( 1 + \frac{G_{Ga}}{28} \left( \frac{14}{4s} \right) \mathfrak{K}_r^4 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $q_p$  = the Planck charge,  $G_{Ga}$  = Gauss's constant,  $s$  = the arc length of the unit lemniscate,  $\mathfrak{K}_r$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_0 = 1.25663706158249 \dots \times 10^{-6} \text{ N/A}^2$	prediction
$\mu_0 = 1.25663706127(20) \times 10^{-6} \text{ N/A}^2$	CODATA 2022, $\sigma = +1.56$
$\mu_0 = 1.25663706212(19) \times 10^{-6} \text{ N/A}^2$	CODATA 2018, $\sigma = -2.83$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.47$

Also called the *permeability of free space*, or the *magnetic constant*.

**electron mass**

268.4338(18)

$$m_e = 2V_{\text{fe}} \left( \frac{m_p^4}{\text{kg}^3} \right) \left( 1 + \left( \frac{14}{4s} \right) \mathfrak{K}_r^4 \boxtimes \right)$$

Where  $V_{\text{fe}}$  = the figure-eight knot hyperbolic volume,  $m_p$  = the Planck mass,  $\text{kg}$  = the kilogram,  $s$  = the arc length of the unit lemniscate,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_e = 9.10938371013637 \dots \times 10^{-31} \text{ kg}$	prediction
$m_e = 9.1093837139(28) \times 10^{-31} \text{ kg}$	CODATA 2022, $\sigma = -1.34$
$m_e = 9.1093837015(28) \times 10^{-31} \text{ kg}$	CODATA 2018, $\sigma = +3.08$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +4.42$

Also listed as the *atomic unit of mass*, and the *natural unit of mass*.

$$m_\tau = \sqrt{\frac{35 L_{LL}}{\varkappa_1^6}} \sqrt{\frac{4s}{2\pi}} \left( \frac{m_p^4}{\text{kg}^3} \right) \left( 1 + \left( \frac{14}{4s} \right) \varkappa_r^4 \boxtimes \right)$$

Where  $L_{LL}$  = the Laplace limit,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $m_p$  = the Planck mass,  $\text{kg}$  = the kilogram,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} m_\tau &= 3.16754546668955 \dots \times 10^{-27} \text{ kg} && \text{prediction} \\ m_\tau &= 3.16754(21) \times 10^{-27} \text{ kg} && \text{CODATA 2022, } \sigma = +0.03 \\ m_\tau &= 3.16754(21) \times 10^{-27} \text{ kg} && \text{CODATA 2018, } \sigma = +0.03 \\ &&& \Delta_{\text{precision}} = +0.00 \\ &&& \Delta_{\text{scaled}} = +0.00 \end{aligned}$$

$$m_{\text{tri}} = \frac{\varkappa_1^{-5}}{18} \sqrt{\frac{4s}{2\pi}} \left( \frac{m_p^4}{\text{kg}^3} \right) \left( 1 + \sqrt{\frac{G_{Ga}}{14}} \left( \frac{18}{2K} \right) \varkappa_r^4 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $m_p$  = the Planck mass,  $\text{kg}$  = the kilogram,  $G_{Ga}$  = Gauss's constant,  $K$  = Catalan's constant,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} m_{\text{tri}} &= 5.00735674868075 \dots \times 10^{-27} \text{ kg} && \text{prediction} \\ m_{\text{tri}} &= 5.0073567512(16) \times 10^{-27} \text{ kg} && \text{CODATA 2022, } \sigma = -1.57 \\ m_{\text{tri}} &= 5.0073567446(15) \times 10^{-27} \text{ kg} && \text{CODATA 2018, } \sigma = +2.72 \\ &&& \Delta_{\text{precision}} = -0.03 \\ &&& \Delta_{\text{scaled}} = +4.40 \end{aligned}$$

$$K_{cd} = \frac{\varkappa_1^{-7}}{f_1} \sqrt{\frac{4s}{2\pi} \left( \frac{t_p^2}{l_p^2 A_{\text{mass}}} \right)} \left( 1 + \sqrt{\frac{G_{Ga}}{14} \left( \frac{18}{2K} \right) \varkappa_r^4 \boxtimes} \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $f_1$  = the frequency corresponding to the peak sensitivity wavelength where energy is maximally converted to light,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $A_{\text{mass}}$  = the atomic mass constant,  $G_{Ga}$  = Gauss's constant,  $K$  = Catalan's constant,  $\varkappa_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$K_{cd} = 6.83002503039578 \dots \times 10^2 \text{ lm/W}$	prediction
$K_{cd} = 6.83 \times 10^2 \text{ lm/W}$	CODATA 2022, 3-digit match
$K_{cd} = 6.83 \times 10^2 \text{ lm/W}$	CODATA 2018, 3-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

$f_1 = 540 \times 10^{12}$  hertz, corresponding to the peak sensitivity wavelength of 555 nm.

**helion mass**

-23.50214(24)

$$m_{\text{he}} = \frac{36}{e\pi} \sqrt{\frac{9s}{4\pi}} (A_{\text{mass}}) \left( 1 - \sqrt{\frac{1}{14}} \left( \frac{x_{\infty}}{2s} \right) \mathfrak{K}_r^4 \boxtimes \right)$$

Where  $e$  = Euler's number,  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $A_{\text{mass}}$  = the atomic mass constant,  $x_{\infty}$  = the 1<sup>st</sup> Foias constant,  $\mathfrak{K}_r$  = the hyperbolic radius constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_{\text{he}} = 5.00641278404447 \dots \times 10^{-27} \text{ kg}$	prediction
$m_{\text{he}} = 5.0064127862(16) \times 10^{-27} \text{ kg}$	CODATA 2022, $\sigma = -1.35$
$m_{\text{he}} = 5.0064127796(15) \times 10^{-27} \text{ kg}$	CODATA 2018, $\sigma = +2.92$
	$\Delta_{\text{precision}} = -0.03$
	$\Delta_{\text{scaled}} = +4.40$

**alpha particle mass**

-469.43891(15)

$$m_{\alpha} = \frac{12}{\omega_2^4} \sqrt{\frac{4s}{2\pi}} (A_{\text{mass}}) \left( 1 - \sqrt{\frac{32}{14}} \left( \frac{2V_{\text{fe}}}{x_{\infty}^2} \right) \mathfrak{K}_r^4 \boxtimes \right)$$

Where  $\omega_2$  = the omega\_2 constant,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $A_{\text{mass}}$  = the atomic mass constant,  $V_{\text{fe}}$  = the figure eight knot hyperbolic volume,  $x_{\infty}$  = the 1<sup>st</sup> Foias constant,  $\mathfrak{K}_r$  = the hyperbolic radian constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_{\alpha} = 6.64465734214075 \dots \times 10^{-27} \text{ kg}$	prediction
$m_{\alpha} = 6.6446573450(21) \times 10^{-27} \text{ kg}$	CODATA 2022, $\sigma = -1.36$
$m_{\alpha} = 6.6446573357(20) \times 10^{-27} \text{ kg}$	CODATA 2018, $\sigma = +3.22$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = +4.65$

$$E_p = \frac{\text{joule}}{\text{GeV}} \left( \frac{l_p^2 m_p}{t_p^2} \right) \left( 1 - \frac{1}{2} \text{Re} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where GeV = the gigaelectron volt,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_p = 1.22088572754077 \dots \times 10^{19} \text{ GeV}$	prediction
$E_p = 1.220890(14) \times 10^{19} \text{ GeV}$	CODATA 2022, $\sigma = -0.31$
$E_p = 1.220890(14) \times 10^{19} \text{ GeV}$	CODATA 2018, $\sigma = -0.31$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**von Klitzing constant**

4.3925(33)

$$R_K = \frac{2\pi}{\varkappa_1^2} \left( \frac{l_p^2 m_p}{t_p q_p^2} \right) \left( 1 + \frac{1}{2} \text{Re} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$R_K = 2.58128074493918 \dots \times 10^4 \Omega$	prediction
$R_K = 2.581280745 \times 10^4 \Omega$	CODATA 2022, 10.63-digit match
$R_K = 2.581280745 \times 10^4 \Omega$	CODATA 2018, 10.63-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

Also listed as the *conventional value of von Klitzing constant*.

## characteristic impedance of vacuum

4.3936(15)

$$Z_0 = 4\pi \left( \frac{l_p^2 m_p}{t_p q_p^2} \right) \left( 1 + \frac{1}{2} \operatorname{Re} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$Z_0 = 3.76730313712033 \dots \times 10^2 \Omega$	prediction
$Z_0 = 3.76730313412(59) \times 10^2 \Omega$	CODATA 2022, $\sigma = +5.09$
$Z_0 = 3.76730313668(57) \times 10^2 \Omega$	CODATA 2018, $\sigma = +0.77$
	$\Delta_{\text{precision}} = -0.05$
	$\Delta_{\text{scaled}} = -4.49$

## Planck electric impedance

4.404(16)

$$Z_p = \left( \frac{l_p^2 m_p}{t_p q_p^2} \right) \left( 1 + \frac{1}{2} \operatorname{Re} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$Z_p = 2.99792458199153 \dots \times 10^1 \Omega$	prediction
$Z_p = 2.99792458(45) \times 10^1 \Omega$	WolframAlpha 2022, $\sigma = +0.00$
$Z_p = 2.99792458(45) \times 10^1 \Omega$	WolframAlpha 2018, $\sigma = +0.00$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**hertz-inverse meter relationship**

3.6106(15)

$$\text{Hz} : \frac{1}{\text{m}} = \left( \frac{t_p}{s l_p} \right) \left( 1 + \frac{1}{2} \text{Im} \left( i^{i^{i^{\dots}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $t_p$  = the Planck time,  $s$  = the second,  $l_p$  = the Planck length,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

Hz : 1/m = 3.33564095322875 ... × 10 <sup>-9</sup> cycles/m	prediction
Hz : 1/m = 3.335640951 × 10 <sup>-9</sup> cycles/m	CODATA 2022, 9.18-digit match
Hz : 1/m = 3.335640951 × 10 <sup>-9</sup> cycles/m	CODATA 2018, 9.18-digit match
	Δ <sub>precision</sub> = +0.00
	Δ <sub>scaled</sub> = +0.00

**atomic unit of velocity**

-3.6170(15)

$$A_{\text{vel}} = \varkappa_1^2 \left( \frac{l_p}{t_p} \right) \left( 1 - \frac{1}{2} \text{Im} \left( i^{i^{i^{\dots}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

A <sub>vel</sub> = 2.18769126392058 ... × 10 <sup>6</sup> m/s	prediction
A <sub>vel</sub> = 2.18769126216(34) × 10 <sup>6</sup> m/s	CODATA 2022, σ = +5.18
A <sub>vel</sub> = 2.18769126364(33) × 10 <sup>6</sup> m/s	CODATA 2018, σ = +3.88
	Δ <sub>precision</sub> = -0.03
	Δ <sub>scaled</sub> = -4.48

**atomic unit of momentum**

-3.6171(15)

$$A_{\text{mom}} = \varkappa_1^2 \left( \frac{l_p m_e}{t_p} \right) \left( 1 - \frac{1}{2} \text{Im} \left( i^{i^{i^{\dots}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} A_{\text{mom}} &= 1.99285191623658 \dots \times 10^{-24} \text{ m} \cdot \text{kg/s} && \text{prediction} \\ A_{\text{mom}} &= 1.99285191545(31) \times 10^{-24} \text{ m} \cdot \text{kg/s} && \text{CODATA 2018, } \sigma = +2.54 \\ A_{\text{mom}} &= 1.99285191410(30) \times 10^{-24} \text{ m} \cdot \text{kg/s} && \text{CODATA 2018, } \sigma = +7.12 \\ &&& \Delta_{\text{precision}} = -0.03 \\ &&& \Delta_{\text{scaled}} = +4.50 \end{aligned}$$

**natural unit of momentum**

-3.6121(30)

$$\mathcal{N}_{\text{mom}} = \left( \frac{l_p m_e}{t_p} \right) \left( 1 - \frac{1}{2} \text{Im} \left( i^{i^{i^{\dots}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \mathcal{N}_{\text{mom}} &= 2.73092453230547 \dots \times 10^{-22} \text{ m} \cdot \text{kg/s} && \text{prediction} \\ \mathcal{N}_{\text{mom}} &= 2.73092453446(85) \times 10^{-22} \text{ m} \cdot \text{kg/s} && \text{CODATA 2022, } \sigma = -2.53 \\ \mathcal{N}_{\text{mom}} &= 2.73092453075(82) \times 10^{-22} \text{ m} \cdot \text{kg/s} && \text{CODATA 2018, } \sigma = +1.90 \\ &&& \Delta_{\text{precision}} = -0.01 \\ &&& \Delta_{\text{scaled}} = +4.52 \end{aligned}$$

**electron volt-kilogram relationship**

7.22329(90)

$$eV : kg = eV \left( \frac{t_p^2}{l_p^2} \right) \left( 1 + Im \left( i^{i^{i^{\cdot}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where  $eV$  = the electron-volt,  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$eV : kg = 1.78266192320573 \dots \times 10^{-36} \text{ kg}$	prediction
$eV : kg = 1.782661921 \times 10^{-36} \text{ kg}$	CODATA 2022, 9.91-digit match
$eV : kg = 1.782661921 \times 10^{-36} \text{ kg}$	CODATA 2018, 9.91-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**joule-kilogram relationship**

7.22398(56)

$$J : kg = \text{joule} \left( \frac{t_p^2}{l_p^2} \right) \left( 1 + Im \left( i^{i^{i^{\cdot}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$J : kg = 1.11265005688553 \dots \times 10^{-17} \text{ kg}$	prediction
$J : kg = 1.112650056 \times 10^{-17} \text{ kg}$	CODATA 2022, 10-digit match
$J : kg = 1.112650056 \times 10^{-17} \text{ kg}$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**joule-atomic mass unit relationship**

7.2242(30)

$$J : A_{\text{mass}} = \text{joule} \left( \frac{t_p^2}{l_p^2 A_{\text{mass}}} \right) \left( 1 + \text{Im} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $A_{\text{mass}}$  = the atomic mass constant,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

J : $A_{\text{mass}} = 6.70053525499264 \dots \times 10^9 \text{ u}$	prediction
J : $A_{\text{mass}} = 6.7005352471(21) \times 10^9 \text{ u}$	CODATA 2022, $\sigma = +3.77$
J : $A_{\text{mass}} = 6.7005352565(20) \times 10^9 \text{ u}$	CODATA 2018, $\sigma = -0.75$
	$\Delta_{\text{precision}} = -0.04$
	$\Delta_{\text{scaled}} = -4.70$

**atomic mass unit-joule relationship**

-7.2242(30)

$$A_{\text{mass}} : J = \left( \frac{l_p^2 A_{\text{mass}}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $A_{\text{mass}}$  = the atomic mass constant,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

A <sub>mass</sub> : J = 1.49241808593480 ... × 10 <sup>-10</sup> J	prediction
A <sub>mass</sub> : J = 1.49241808768(46) × 10 <sup>-10</sup> J	CODATA 2022, $\sigma = -3.79$
A <sub>mass</sub> : J = 1.49241808560(45) × 10 <sup>-10</sup> J	CODATA 2018, $\sigma = +0.74$
	$\Delta_{\text{precision}} = -0.00$
	$\Delta_{\text{scaled}} = +4.62$

Also listed as the *atomic mass constant energy equivalent*.

**natural unit of momentum in MeV/c**

-7.2227(29)

$$\mathcal{N}_{\text{mom}}^{\cdot} = \frac{\text{joule}}{\text{MeV}} \left( \frac{s l_p^2 m_e}{t_p^2 m} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where MeV = the megaelectron-volt, s = the second,  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $t_p$  = the Planck time, m = the meter,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \mathcal{N}_{\text{mom}}^{\cdot} &= 5.10998950028076 \dots \times 10^{-1} \text{ MeV}/c && \text{prediction} \\ \mathcal{N}_{\text{mom}}^{\cdot} &= 5.1099895069(16) \times 10^{-1} \text{ MeV}/c && \text{CODATA 2022, } \sigma = -4.14 \\ \mathcal{N}_{\text{mom}}^{\cdot} &= 5.1099895000(15) \times 10^{-1} \text{ MeV}/c && \text{CODATA 2018, } \sigma = +0.19 \\ &&& \Delta_{\text{precision}} = -0.02 \\ &&& \Delta_{\text{scaled}} = +4.60 \end{aligned}$$

**atomic mass constant energy equivalent in MeV**

-7.2227(30)

$$E_{A_{\text{mass}}}^{\cdot} = \frac{1}{\text{MeV}} \left( \frac{l_p^2 A_{\text{mass}}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where MeV = the megaelectron-volt,  $l_p$  = the Planck length,  $A_{\text{mass}}$  = the atomic mass constant,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} E_{A_{\text{mass}}}^{\cdot} &= 9.31494102497143 \dots \times 10^2 \text{ MeV} && \text{prediction} \\ E_{A_{\text{mass}}}^{\cdot} &= 9.3149410372(29) \times 10^2 \text{ MeV} && \text{CODATA 2022, } \sigma = -4.22 \\ E_{A_{\text{mass}}}^{\cdot} &= 9.3149410242(28) \times 10^2 \text{ MeV} && \text{CODATA 2018, } \sigma = -0.25 \\ &&& \Delta_{\text{precision}} = -0.01 \\ &&& \Delta_{\text{scaled}} = +4.64 \end{aligned}$$

**kilogram-joule relationship**

-7.22398(56)

$$\text{kg} : \text{J} = \left( \frac{l_p^2 \text{ kg}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length, kg = the kilogram,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

kg : J = 8.98755178064359 ... × 10 <sup>16</sup> J	prediction
kg : J = 8.987551787 × 10 <sup>16</sup> J	CODATA 2022, 9.15-digit match
kg : J = 8.987551787 × 10 <sup>16</sup> J	CODATA 2018, 9.15-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**kilogram-electron volt relationship**

-7.22329(90)

$$\text{kg} : \text{eV} = \frac{\text{joule}}{\text{eV}} \left( \frac{l_p^2 \text{ kg}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where eV = the electron-volt,  $l_p$  = the Planck length, kg = the kilogram,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

Kg : eV = 5.60958859883646 ... × 10 <sup>35</sup> eV	prediction
kg : eV = 5.609588603 × 10 <sup>35</sup> eV	CODATA 2022, 9.13-digit match
kg : eV = 5.609588603 × 10 <sup>35</sup> eV	CODATA 2018, 9.13-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**neutron-proton mass difference energy equivalent**

-7.9(35)

$$E_{\Delta} = (m_n - m_+) \left( \frac{l_p^2}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $m_n$  = the neutron mass,  $m_+$  = the proton mass,  $l_p$  = the Planck length,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_{\Delta} = 2.07214623577065 \dots \times 10^{-13} \text{ J}$	prediction
$E_{\Delta} = 2.07214712(60) \times 10^{-13} \text{ J}$	CODATA 2022, $\sigma = -1.47$
$E_{\Delta} = 2.07214689(74) \times 10^{-13} \text{ J}$	CODATA 2018, $\sigma = -0.88$
	$\Delta_{\text{precision}} = +0.09$
	$\Delta_{\text{scaled}} = +0.31$

**neutron-proton mass difference energy equivalent in MeV**

-7.9(52)

$$\dot{E}_{\Delta} = \frac{\text{joule}}{\text{MeV}} (m_n - m_+) \left( \frac{l_p^2}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where, MeV = the megaelectron-volt,  $m_n$  = the neutron mass,  $m_+$  = the proton mass,  $l_p$  = the Planck length,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\dot{E}_{\Delta} = 1.29333195323950 \dots \text{ MeV}$	prediction
$\dot{E}_{\Delta} = 1.29333251(38) \text{ MeV}$	CODATA 2022, $\sigma = -1.47$
$\dot{E}_{\Delta} = 1.29333236(46) \text{ MeV}$	CODATA 2018, $\sigma = -0.88$
	$\Delta_{\text{precision}} = +0.11$
	$\Delta_{\text{scaled}} = +0.32$

**natural unit of energy in MeV**

-7.2227(29)

$$\dot{E}_e = \frac{\text{joule}}{\text{MeV}} \left( \frac{l_p^2 m_e}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\dots}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where MeV = the megaelectron-volt,  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\dot{E}_e = 5.10998950028076 \dots \times 10^{-1} \text{ MeV}$	prediction
$\dot{E}_e = 5.1099895069(16) \times 10^{-1} \text{ MeV}$	CODATA 2022, $\sigma = -4.14$
$\dot{E}_e = 5.1099895000(15) \times 10^{-1} \text{ MeV}$	CODATA 2018, $\sigma = +0.19$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = +4.60$

Also listed as the *electron mass energy equivalent in MeV*.

**muon mass energy equivalent in MeV**

-7.20(21)

$$\dot{E}_\mu = \frac{\text{joule}}{\text{MeV}} \left( \frac{l_p^2 m_\mu}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\dots}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where MeV = the megaelectron-volt,  $l_p$  = the Planck length,  $m_\mu$  = the muon mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\dot{E}_\mu = 1.05658374731846 \dots \times 10^2 \text{ MeV}$	prediction
$\dot{E}_\mu = 1.056583755(23) \times 10^2 \text{ MeV}$	CODATA 2022, $\sigma = -0.33$
$\dot{E}_\mu = 1.056583755(23) \times 10^2 \text{ MeV}$	CODATA 2018, $\sigma = -0.33$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**proton mass energy equivalent in MeV**

-7.2229(31)

$$E_+ = \frac{\text{joule}}{\text{MeV}} \left( \frac{l_p^2 m_+}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\dots}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where MeV = the megaelectron-volt,  $l_p$  = the Planck length,  $m_+$  = the proton mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_+ = 9.38272088205557 \dots \times 10^2 \text{ MeV}$	prediction
$E_+ = 9.3827208943(29) \times 10^2 \text{ MeV}$	CODATA 2022, $\sigma = -4.22$
$E_+ = 9.3827208816(29) \times 10^2 \text{ MeV}$	CODATA 2018, $\sigma = +0.16$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = +4.37$

**neutron mass energy equivalent in MeV**

-7.2239(58)

$$E_n = \frac{\text{joule}}{\text{MeV}} \left( \frac{l_p^2 m_n}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\dots}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where MeV = the megaelectron-volt,  $l_p$  = the Planck length,  $m_n$  = the neutron mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_n = 9.39565420158796 \dots \times 10^2 \text{ MeV}$	prediction
$E_n = 9.3956542194(48) \times 10^2 \text{ MeV}$	CODATA 2022, $\sigma = -3.71$
$E_n = 9.3956542052(54) \times 10^2 \text{ MeV}$	CODATA 2018, $\sigma = -0.67$
	$\Delta_{\text{precision}} = +0.08$
	$\Delta_{\text{scaled}} = +2.63$

**tau energy equivalent**

-23.5(722)

$$\dot{E}_\tau = \frac{\text{joule}}{\text{MeV}} \left( \frac{l_p^2 m_\tau}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where MeV = the megaelectron-volt,  $l_p$  = the Planck length,  $m_\tau$  = the tau mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\dot{E}_\tau = 1.77686269362378 \dots \times 10^3 \text{ MeV}$	prediction
$\dot{E}_\tau = 1.77686(12) \times 10^3 \text{ MeV}$	CODATA 2022, $\sigma = +0.02$
$\dot{E}_\tau = 1.77686(12) \times 10^3 \text{ MeV}$	CODATA 2018, $\sigma = +0.02$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**deuteron mass energy equivalent in MeV**

-7.2225(30)

$$\dot{E}_{\text{de}} = \frac{\text{joule}}{\text{MeV}} \left( \frac{l_p^2 m_{\text{de}}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where MeV = the megaelectron-volt,  $l_p$  = the Planck length,  $m_{\text{de}}$  = the deuteron mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\dot{E}_{\text{de}} = 1.87561294255340 \dots \times 10^3 \text{ MeV}$	prediction
$\dot{E}_{\text{de}} = 1.87561294500(58) \times 10^3 \text{ MeV}$	CODATA 2022, $\sigma = -4.22$
$\dot{E}_{\text{de}} = 1.87561294257(57) \times 10^3 \text{ MeV}$	CODATA 2018, $\sigma = +0.03$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = +4.26$

**helion mass energy equivalent in MeV**

-7.2228(30)

$$E_{\text{he}}^{\cdot} = \frac{\text{joule}}{\text{MeV}} \left( \frac{l_p^2 m_{\text{he}}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where MeV = the megaelectron-volt,  $l_p$  = the Planck length,  $m_{\text{he}}$  = the helion mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_{\text{he}}^{\cdot} = 2.80839160744450 \dots \times 10^3 \text{ MeV}$	prediction
$E_{\text{he}}^{\cdot} = 2.80839161112(88) \times 10^3 \text{ MeV}$	CODATA 2022, $\sigma = -4.18$
$E_{\text{he}}^{\cdot} = 2.80839160743(85) \times 10^3 \text{ MeV}$	CODATA 2018, $\sigma = +0.02$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = +4.34$

**triton mass energy equivalent in MeV**

-7.2221(30)

$$E_{\text{tri}}^{\cdot} = \frac{\text{joule}}{\text{MeV}} \left( \frac{l_p^2 m_{\text{tri}}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where MeV = the megaelectron-volt,  $l_p$  = the Planck length,  $m_{\text{tri}}$  = the triton mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_{\text{tri}}^{\cdot} = 2.80892113277064 \dots \times 10^3 \text{ MeV}$	prediction
$E_{\text{tri}}^{\cdot} = 2.80892113668(88) \times 10^3 \text{ MeV}$	CODATA 2018, $\sigma = -4.44$
$E_{\text{tri}}^{\cdot} = 2.80892113298(85) \times 10^3 \text{ MeV}$	CODATA 2018, $\sigma = -0.25$
	$\Delta_{\text{precision}} = -0.03$
	$\Delta_{\text{scaled}} = +4.35$

# alpha particle mass energy equivalent in MeV

-7.2235(30)

$$E_{\alpha} = \frac{\text{joule}}{\text{MeV}} \left( \frac{l_p^2 m_{\alpha}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\dots}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where MeV = the megaelectron-volt,  $l_p$  = the Planck length,  $m_{\alpha}$  = the alpha particle mass,  $l_p$  = the Planck length,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$E_{\alpha} = 3.72737940696477 \dots \times 10^3 \text{ MeV}$$

prediction

$$E_{\alpha} = 3.7273794118(12) \times 10^3 \text{ MeV}$$

CODATA 2022,  $\sigma = -4.03$

$$E_{\alpha} = 3.7273794066(11) \times 10^3 \text{ MeV}$$

CODATA 2018,  $\sigma = +0.33$

$$\Delta_{\text{precision}} = -0.03$$

$$\Delta_{\text{scaled}} = +4.72$$

**electron mass energy equivalent**

-7.2241(30)

$$E_e = \left( \frac{l_p^2 m_e}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\dots}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_e = 8.18710577846019 \dots \times 10^{-14} \text{ J}$	prediction
$E_e = 8.1871057880(26) \times 10^{-14} \text{ J}$	CODATA 2018, $\sigma = -3.67$
$E_e = 8.1871057769(25) \times 10^{-14} \text{ J}$	CODATA 2018, $\sigma = +0.62$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = +4.44$

Also listed as the *natural unit of energy*.

**muon mass energy equivalent**

-7.19(24)

$$E_\mu = \left( \frac{l_p^2 m_\mu}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\dots}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_\mu$  = the muon mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_\mu = 1.69283379205041 \dots \times 10^{-11} \text{ J}$	prediction
$E_\mu = 1.692833804(38) \times 10^{-11} \text{ J}$	CODATA 2022, $\sigma = -0.31$
$E_\mu = 1.692833804(38) \times 10^{-11} \text{ J}$	CODATA 2018, $\sigma = -0.31$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**proton mass energy equivalent**

-7.2242(31)

$$E_+ = \left( \frac{l_p^2 m_+}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_+$  = the proton mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_+ = 1.50327761626390 \dots \times 10^{-10} \text{ J}$	prediction
$E_+ = 1.50327761802(47) \times 10^{-10} \text{ J}$	CODATA 2022, $\sigma = -3.74$
$E_+ = 1.50327761598(46) \times 10^{-10} \text{ J}$	CODATA 2018, $\sigma = +0.62$
	$\Delta_{\text{precision}} = -0.04$
	$\Delta_{\text{scaled}} = +4.43$

**neutron mass energy equivalent**

-7.2253(57)

$$E_n = \left( \frac{l_p^2 m_n}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_n$  = the neutron mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_n = 1.50534976249967 \dots \times 10^{-10} \text{ J}$	prediction
$E_n = 1.50534976514(76) \times 10^{-10} \text{ J}$	CODATA 2022, $\sigma = -3.47$
$E_n = 1.50534976287(86) \times 10^{-10} \text{ J}$	CODATA 2018, $\sigma = -0.43$
	$\Delta_{\text{precision}} = +0.07$
	$\Delta_{\text{scaled}} = +2.63$

**tau mass energy equivalent**

-40(668)

$$E_{\tau} = \left( \frac{l_p^2 m_{\tau}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_{\tau}$  = the tau mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_{\tau} = 2.84684788994152 \dots \times 10^{-10} \text{ J}$	prediction
$E_{\tau} = 2.84684(19) \times 10^{-10} \text{ J}$	CODATA 2022, $\sigma = +0.04$
$E_{\tau} = 2.84684(19) \times 10^{-10} \text{ J}$	CODATA 2018, $\sigma = +0.04$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**deuteron mass energy equivalent**

-7.2239(30)

$$E_{\text{de}} = \left( \frac{l_p^2 m_{\text{de}}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_{\text{de}}$  = the deuteron mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_{\text{de}} = 3.00506323139998 \dots \times 10^{-10} \text{ J}$	prediction
$E_{\text{de}} = 3.00506323491(94) \times 10^{-10} \text{ J}$	CODATA 2022, $\sigma = -3.73$
$E_{\text{de}} = 3.00506323102(91) \times 10^{-10} \text{ J}$	CODATA 2018, $\sigma = +0.42$
	$\Delta_{\text{precision}} = -0.04$
	$\Delta_{\text{scaled}} = +4.27$

**helion mass energy equivalent**

-7.2243(31)

$$E_{\text{he}} = \left( \frac{l_p^2 m_{\text{he}}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\dots}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_{\text{he}}$  = the helion mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_{\text{he}} = 4.49953941318757 \dots \times 10^{-10} \text{ J}$	prediction
$E_{\text{he}} = 4.4995394185(14) \times 10^{-10} \text{ J}$	CODATA 2022, $\sigma = -3.79$
$E_{\text{he}} = 4.4995394125(14) \times 10^{-10} \text{ J}$	CODATA 2018, $\sigma = +0.49$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +4.29$

**triton mass energy equivalent**

-7.2234(31)

$$E_{\text{tri}} = \left( \frac{l_p^2 m_{\text{tri}}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\dots}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_{\text{tri}}$  = the triton mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_{\text{tri}} = 4.50038780629234 \dots \times 10^{-10} \text{ J}$	prediction
$E_{\text{tri}} = 4.5003878119(14) \times 10^{-10} \text{ J}$	CODATA 2022, $\sigma = -4.01$
$E_{\text{tri}} = 4.5003878060(14) \times 10^{-10} \text{ J}$	CODATA 2018, $\sigma = +0.21$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +4.21$

**alpha particle mass energy equivalent**

-7.2247(30)

$$E_{\alpha} = \left( \frac{l_p^2 m_{\alpha}}{t_p^2} \right) \left( 1 - \text{Im} \left( i^{i^{i^{\cdot}}} \right) \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_{\alpha}$  = the alpha particle mass,  $t_p$  = the Planck time,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$E_{\alpha} = 5.97192019271236 \dots \times 10^{-10} \text{ J}$$

$$E_{\alpha} = 5.9719201997(19) \times 10^{-10} \text{ J}$$

$$E_{\alpha} = 5.9719201914(18) \times 10^{-10} \text{ J}$$

prediction

CODATA 2022,  $\sigma = -3.68$ CODATA 2018,  $\sigma = +0.73$  $\Delta_{\text{precision}} = -0.01$  $\Delta_{\text{scaled}} = +4.61$

**deuteron g factor**

-181.339(25)

$$g_{de} = \frac{2\pi}{8K} \left( 1 - 8 \left( \frac{2\pi}{W_{We}} \right) \varkappa_1 \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $W_{We}$  = the Weierstrass constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$g_{de} = 8.57438235960775 \dots \times 10^{-1}$	prediction
$g_{de} = 8.574382335(22) \times 10^{-1}$	CODATA 2022, $\sigma = +1.12$
$g_{de} = 8.574382338(22) \times 10^{-1}$	CODATA 2018, $\sigma = +0.98$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.14$

**deuteron magnetic moment to nuclear magneton ratio**

-181.339(26)

$$\frac{\mu_{de}}{\mu_N} = \frac{2\pi}{8K} \left( 1 - 8 \left( \frac{2\pi}{W_{We}} \right) \varkappa_1 \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $W_{We}$  = the Weierstrass constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_{de}/\mu_N = 8.57438235960775 \times 10^{-1}$	prediction
$\mu_{de}/\mu_N = 8.574382335(22) \times 10^{-1}$	CODATA 2022, $\sigma = +1.12$
$\mu_{de}/\mu_N = 8.574382338(22) \times 10^{-1}$	CODATA 2018, $\sigma = +0.98$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.14$

**deuteron magnetic moment to Bohr magneton ratio** -1201.313(25)

$$\frac{\mu_{de}}{\mu_B} = \frac{2\pi}{8K} \left( \frac{m_e}{m_+} \right) \left( 1 - 8 \left( \frac{2\pi}{W_{We}} \right) \mathfrak{K}_1 \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $W_{We}$  = the Weierstrass constant,  $\mathfrak{K}_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_{de}/\mu_B = 4.66975458182757 \times 10^{-4}$	prediction
$\mu_{de}/\mu_B = 4.669754568(12) \times 10^{-4}$	CODATA 2022, $\sigma = +1.15$
$\mu_{de}/\mu_B = 4.669754570(12) \times 10^{-4}$	CODATA 2018, $\sigma = +0.99$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.17$

**muon magnetic moment**

717.99(23)

$$\mu_\mu = -\frac{\varkappa_1^2 \varkappa_2^2}{P_{up}} \left( \frac{l_p^2 q_p m_p}{t_p m_\mu} \right) \left( 1 + \frac{4\pi}{\mathcal{L}_{Li}} (\pi - 1) \varkappa_1^2 \varkappa_3^2 \varkappa_4^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_\mu$  = the muon mass,  $\pi$  = Archimedes' constant,  $\mathcal{L}_{Li}$  = Liouville's constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_\mu = -4.49044833035195 \dots \times 10^{-26} \text{ J/T}$	prediction
$\mu_\mu = -4.49044830(10) \times 10^{-26} \text{ J/T}$	CODATA 2022, $\sigma = -0.30$
$\mu_\mu = -4.49044830(10) \times 10^{-26} \text{ J/T}$	CODATA 2018, $\sigma = -0.30$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**electron magnetic moment anomaly**

-650.6071858(18)

$$a_e = \frac{2 \varkappa_1 \varkappa_2^2}{P_{up}} - 1 \left( 1 - \frac{35}{\mathcal{L}_{Li}} \left( \frac{\sqrt{2}}{V_{fe}} \right) \varkappa_1^2 \varkappa_3^2 \varkappa_4^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $\mathcal{L}_{Li}$  = Liouville's constant,  $V_{fe}$  = the figure-eight knot hyperbolic volume,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$a_e = 1.15965218025405 \dots \times 10^{-3}$	prediction
$a_e = 1.15965218046(18) \times 10^{-3}$	CODATA 2022, $\sigma = -1.14$
$a_e = 1.15965218128(18) \times 10^{-3}$	CODATA 2018, $\sigma = -5.70$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -4.56$

**muon magnetic moment anomaly**

-713.2916(41)

$$a_\mu = \frac{2 \kappa_1 \kappa_2^2}{P_{up}} - 1 \left( 1 - \frac{!5}{\sqrt{\mathcal{L}_{Li}}} (2K) \kappa_1^2 \kappa_3^2 \kappa_4^2 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $!n$  = the derangement function,  $\mathcal{L}_{Li}$  = Liouville's constant,  $K$  = Catalan's constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$a_\mu = 1.16592060110720 \dots \times 10^{-3}$	prediction
$a_\mu = 1.16592062(41) \times 10^{-3}$	CODATA 2022, $\sigma = -0.05$
$a_\mu = 1.16592089(63) \times 10^{-3}$	CODATA 2018, $\sigma = -0.46$
	$\Delta_{\text{precision}} = +0.18$
	$\Delta_{\text{scaled}} = -0.43$

**muon g factor**

712.5140(65)

$$g_\mu = -\frac{4 \kappa_1 \kappa_2^2}{P_{up}} \left( 1 + \zeta(2)^{1/3} \left( \frac{35}{C_d} \right) \kappa_1^3 \kappa_3^3 \kappa_4^3 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $\zeta(x)$  = the Riemann zeta function,  $C_d$  = the domino tiling constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$g_\mu = -2.00233184172840 \dots$	prediction
$g_\mu = -2.00233184123(82)$	CODATA 2022, $\sigma = -0.61$
$g_\mu = -2.0023318418(13)$	CODATA 2018, $\sigma = +0.06$
	$\Delta_{\text{precision}} = +1.80$
	$\Delta_{\text{scaled}} = +0.50$

**muon magnetic moment to Bohr magneton ratio**

712.48(22)

$$\frac{\mu_\mu}{\mu_B} = -\frac{2 \mathfrak{K}_1 \mathfrak{K}_2^2}{P_{up}} \left( \frac{m_e}{m_\mu} \right) \left( 1 + \zeta(2)^{1/3} \left( \frac{35}{C_d} \right) \mathfrak{K}_1^3 \mathfrak{K}_3^3 \mathfrak{K}_4^3 \boxtimes \right)$$

Where  $\mathfrak{K}_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_e$  = the electron mass,  $m_\mu$  = the muon mass,  $\zeta(x)$  = the Riemann zeta function,  $C_d$  = the domino tiling constant,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_\mu/\mu_B = -4.84197050791187 \dots \times 10^{-3}$	prediction
$\mu_\mu/\mu_B = -4.84197048(11) \times 10^{-3}$	CODATA 2022, $\sigma = -0.25$
$\mu_\mu/\mu_B = -4.84197047(11) \times 10^{-3}$	CODATA 2018, $\sigma = -0.34$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.09$

**muon magnetic moment to nuclear magneton ratio**

712.48(22)

$$\frac{\mu_\mu}{\mu_N} = -\frac{2 \mathfrak{K}_1 \mathfrak{K}_2^2}{P_{up}} \left( \frac{m_+}{m_\mu} \right) \left( 1 + \zeta(2)^{1/3} \left( \frac{35}{C_d} \right) \mathfrak{K}_1^3 \mathfrak{K}_3^3 \mathfrak{K}_4^3 \boxtimes \right)$$

Where  $\mathfrak{K}_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_+$  = the proton mass,  $m_\mu$  = the muon mass,  $\zeta(x)$  = the Riemann zeta function,  $C_d$  = the domino tiling constant,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_\mu/\mu_N = -8.89059709269183 \dots$	prediction
$\mu_\mu/\mu_N = -8.89059704(20)$	CODATA 2022, $\sigma = -0.26$
$\mu_\mu/\mu_N = -8.89059703(20)$	CODATA 2018, $\sigma = -0.31$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.05$

**molar gas constant**

-236.35823(30)

$$R = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{l_p^2 C}{t_p^2 m q_p T_p} \right) \left( 1 - \frac{8}{C_d} \kappa_1^4 \kappa_3^4 \kappa_4^4 \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $C$  = the coulomb,  $t_p$  = the Planck time,  $m$  = the meter,  $q_p$  = the Planck charge,  $T_p$  = The Planck temperature,  $C_d$  = the domino tiling constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$R = 8.31446261978165 \dots$ J/mol · K	prediction
$R = 8.314462618$ J/mol · K	CODATA 2022, 9.67-digit match
$R = 8.314462618$ J/mol · K	CODATA 2018, 9.67-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**molar volume of ideal gas (273.15 K, 100 kPa)**

-236.357(02)

$$V_{m_0} = \frac{6}{e^\gamma} \kappa_1^2 \frac{T_0}{p_0} \left( \frac{l_p^2 C}{t_p^2 m q_p T_p} \right) \left( 1 - \frac{8}{C_d} \kappa_1^4 \kappa_3^4 \kappa_4^4 \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $T_0 = 273.150000000000$  K,  $p_0 = 100.000000000000$  kPa,  $l_p$  = the Planck length,  $C$  = the coulomb,  $t_p$  = the Planck time,  $m$  = the meter,  $q_p$  = the Planck charge,  $T_p$  = the Planck temperature,  $C_d$  = the domino tiling constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$V_{m_0} = 2.27109546459336 \dots \times 10^{-2}$ m <sup>3</sup> /mol	prediction
$V_{m_0} = 2.271095464 \times 10^{-2}$ m <sup>3</sup> /mol	CODATA 2022, 10-digit match
$V_{m_0} = 2.271095464 \times 10^{-2}$ m <sup>3</sup> /mol	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**molar volume of ideal gas (273.15 K, 101.325 kPa)** −236.359(22)

$$V_{m_1} = \frac{6}{e^\gamma} \kappa_1^2 \frac{T_0}{p_1} \left( \frac{l_p^2 C}{t_p^2 m q_p T_p} \right) \left( 1 - \frac{8}{C_d} \kappa_1^4 \kappa_3^4 \kappa_4^4 \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $T_0 = 273.150000000000\text{ K}$ ,  $p_1 = 101.325000000000\text{ kPa}$ ,  $l_p$  = the Planck length,  $C$  = the coulomb,  $t_p$  = the Planck time,  $m$  = the meter,  $q_p$  = the Planck charge,  $T_p$  = the Planck temperature,  $C_d$  = the domino tiling constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$V_{m_1} = 2.24139695494040 \dots \times 10^{-2} \text{ m}^3/\text{mol}$	prediction
$V_{m_1} = 2.241396954 \times 10^{-2} \text{ m}^3/\text{mol}$	CODATA 2022, 10-digit match
$V_{m_1} = 2.241396954 \times 10^{-2} \text{ m}^3/\text{mol}$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**electron g factor** 649.8958153(18)

$$g_e = -\frac{4 \kappa_1 \kappa_2^2}{P_{up}} \left( 1 + C_{\text{CFP}}^2 \left( \frac{32}{D_d} \right) \kappa_1^6 \kappa_3^6 \kappa_4^6 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $D_d$  = the dimer constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$g_e = -2.00231930436065 \dots$	prediction
$g_e = -2.00231930436092(36)$	CODATA 2022, $\sigma = +0.74$
$g_e = -2.00231930436256(35)$	CODATA 2018, $\sigma = -5.46$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = -4.69$

**electron magnetic moment to Bohr magneton ratio**      649.8958153(18)

$$\frac{\mu_e}{\mu_B} = -\frac{2 \varkappa_1 \varkappa_2^2}{P_{up}} \left( 1 + C_{CFP}^2 \left( \frac{32}{D_d} \right) \varkappa_1^6 \varkappa_3^6 \varkappa_4^6 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $C_{CFP}$  = the fixed point of the hyperbolic cotangent,  $D_d$  = the dimer constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_e/\mu_B = -1.00115965218033 \dots$	prediction
$\mu_e/\mu_B = -1.00115965218046(18)$	CODATA 2022, $\sigma = +0.74$
$\mu_e/\mu_B = -1.00115965218128(18)$	CODATA 2018, $\sigma = -5.28$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -4.56$

**electron magnetic moment to nuclear magneton ratio**      649.89584(17)

$$\frac{\mu_e}{\mu_N} = -\frac{2 \varkappa_1 \varkappa_2^2}{P_{up}} \left( \frac{m_+}{m_e} \right) \left( 1 + C_{CFP}^2 \left( \frac{32}{D_d} \right) \varkappa_1^6 \varkappa_3^6 \varkappa_4^6 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_+$  = the proton mass,  $m_e$  = the electron mass,  $C_{CFP}$  = the fixed point of the hyperbolic cotangent,  $D_d$  = the dimer constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_e/\mu_N = -1.83828197186466 \dots \times 10^3$	prediction
$\mu_e/\mu_N = -1.838281971877(32) \times 10^3$	CODATA 2022, $\sigma = +0.39$
$\mu_e/\mu_N = -1.83828197188(11) \times 10^3$	CODATA 2018, $\sigma = -0.14$
	$\Delta_{\text{precision}} = +0.54$
	$\Delta_{\text{scaled}} = -0.03$

**electron magnetic moment**

655.4130(32)

$$\mu_e = -\frac{\kappa_1^2 \kappa_2^2}{P_{up}} \left( \frac{l_p^2 q_p m_p}{t_p m_e} \right) \left( 1 + \sqrt{7} \left( \frac{8s}{4\pi} \right) \kappa_1^8 \kappa_3^8 \kappa_4^8 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_e$  = the electron mass,  $\pi$  = Archimedes' constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_e = -9.28476469207109 \dots \times 10^{-24} \text{ J/T}$	prediction
$\mu_e = -9.2847646917(29) \times 10^{-24} \text{ J/T}$	CODATA 2022, $\sigma = -0.13$
$\mu_e = -9.2847647043(28) \times 10^{-24} \text{ J/T}$	CODATA 2018, $\sigma = -4.37$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.50$

**electron gyromagnetic ratio**

650.2658(31)

$$\gamma_e = \frac{2 \kappa_1^2 \kappa_2^2}{P_{up}} \left( \frac{q_p}{m_e} \right) \left( 1 + 7 \left( \frac{3s}{4\pi} \right) \kappa_1^8 \kappa_3^8 \kappa_4^8 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $q_p$  = the Planck charge,  $m_e$  = the electron mass,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\gamma_e = 1.76085962907620 \dots \times 10^{11} \text{ 1/s} \cdot \text{T}$	prediction
$\gamma_e = 1.76085962784(55) \times 10^{11} \text{ 1/s} \cdot \text{T}$	CODATA 2022, $\sigma = +2.25$
$\gamma_e = 1.76085963023(53) \times 10^{11} \text{ 1/s} \cdot \text{T}$	CODATA 2018, $\sigma = -2.17$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = 4.51$

$$\dot{\gamma}_e = \frac{\varkappa_1^2 \varkappa_2^2}{\pi P_{up}} \frac{1}{\text{MHz}} \left( \frac{q_p}{s m_e} \right) \left( 1 + 7 \left( \frac{3s}{4\pi} \right) \varkappa_1^8 \varkappa_3^8 \varkappa_4^8 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $P_{up}$  = the universal parabolic constant, MHz = the mega hertz,  $q_p$  = the Planck charge,  $s$  = the second,  $m_e$  = the electron mass,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\dot{\gamma}_e = 2.80249514058438 \dots \times 10^4 \text{ MHz/T}$	prediction
$\dot{\gamma}_e = 2.80249513861(87) \times 10^4 \text{ MHz/T}$	CODATA 2022, $\sigma = +2.27$
$\dot{\gamma}_e = 2.80249514242(85) \times 10^4 \text{ MHz/T}$	CODATA 2018, $\sigma = -2.16$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.48$

**shielded proton gyromagnetic ratio in MHz/T**

-580.805(41)

$$\dot{\gamma}'_+ = \frac{\varkappa_1^2 \varkappa_2^2}{\pi S} \frac{1}{\text{MHz}} \left( \frac{q_p}{s m_+} \right) \left( 1 - \sqrt{35} \left( \frac{7}{S} \right) \varkappa_2 \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $S$  = Sierpiński's constant, MHz = the megahertz,  $q_p$  = the Planck charge,  $s$  = the second,  $m_+$  = the proton mass,  $s$  = the arc length of the unit lemniscate,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\dot{\gamma}'_+ = 4.25763852810155 \dots \times 10^1 \text{ MHz/T}$	prediction
$\dot{\gamma}'_+ = 4.257638543(17) \times 10^1 \text{ MHz/T}$	CODATA 2022, $\sigma = -0.88$
$\dot{\gamma}'_+ = 4.257638474(46) \times 10^1 \text{ MHz/T}$	CODATA 2018, $\sigma = +1.18$
	$\Delta_{\text{precision}} = +0.44$
	$\Delta_{\text{scaled}} = +1.50$

**shielded proton gyromagnetic ratio**

-580.799(41)

$$\gamma'_+ = \frac{2 \varkappa_1^2 \varkappa_2^2}{S} \left( \frac{q_p}{m_+} \right) \left( 1 - \sqrt{35} \left( \frac{7}{S} \right) \varkappa_2 \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $S$  = Sierpiński's constant,  $q_p$  = the Planck charge,  $m_+$  = the proton mass,  $s$  = the arc length of the unit lemniscate,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\gamma'_+ = 2.67515318430494 \dots \times 10^8 \text{ 1/s} \cdot \text{T}$	prediction
$\gamma'_+ = 2.675153194(11) \times 10^8 \text{ 1/s} \cdot \text{T}$	CODATA 2022, $\sigma = -0.88$
$\gamma'_+ = 2.675153151(29) \times 10^8 \text{ 1/s} \cdot \text{T}$	CODATA 2018, $\sigma = -0.33$
	$\Delta_{\text{precision}} = +0.53$
	$\Delta_{\text{scaled}} = +1.48$

## shielded proton magnetic moment

-575.655(41)

$$\mu_+' = \frac{\varkappa_1^2 \varkappa_2^2}{S} \left( \frac{l_p^2 q_p m_p}{t_p m_+} \right) \left( 1 + \frac{\sqrt{3 \cdot 35}}{M} \left( \frac{7}{s} \right) \varkappa_2 \varkappa_3 \varkappa_4 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $S$  = Sierpiński's constant,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_+$  = the proton mass,  $M$  = the Madelung constant,  $s$  = the arc length of the unit lemniscate,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_+' = 1.41057057939798 \dots \times 10^{-26} \text{ J/T}$	prediction
$\mu_+' = 1.4105705830(58) \times 10^{-26} \text{ J/T}$	CODATA 2022, $\sigma = -0.62$
$\mu_+' = 1.410570560(15) \times 10^{-26} \text{ J/T}$	CODATA 2018, $\sigma = +1.29$
	$\Delta_{\text{precision}} = +0.43$
	$\Delta_{\text{scaled}} = +1.53$

**proton magnetic moment to Bohr magneton ratio** -324.4677(31)

$$\frac{\mu_+}{\mu_B} = \frac{2 \kappa_1 \kappa_2^2}{S} \left( \frac{m_e}{m_+} \right) \left( 1 - 6 \left( \frac{2\pi}{8K} \right)^2 \kappa_2 \kappa_3 \kappa_4 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $S$  = Sierpiński's constant,  $m_e$  = the electron mass,  $m_+$  = the proton mass,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_+/\mu_B = 1.52103220255243 \dots \times 10^{-3}$	prediction
$\mu_+/\mu_B = 1.52103220230(45) \times 10^{-3}$	CODATA 2022, $\sigma = +0.56$
$\mu_+/\mu_B = 1.52103220230(46) \times 10^{-3}$	CODATA 2018, $\sigma = +0.55$
	$\Delta_{\text{precision}} = +0.01$
	$\Delta_{\text{scaled}} = +0.00$

**proton magnetic moment to nuclear magneton ratio** -324.4678(29)

$$\frac{\mu_+}{\mu_N} = \frac{2 \kappa_1 \kappa_2^2}{S} \left( 1 - 6 \left( \frac{2\pi}{8K} \right)^2 \kappa_2 \kappa_3 \kappa_4 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $S$  = Sierpiński's constant,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_+/\mu_N = 2.79284734506470 \dots$	prediction
$\mu_+/\mu_N = 2.79284734463(82)$	CODATA 2022, $\sigma = +0.53$
$\mu_+/\mu_N = 2.79284734463(82)$	CODATA 2018, $\sigma = +0.53$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**proton g factor**

-324.4677(18)

$$g_+ = \frac{4 \kappa_1 \kappa_2^2}{S} \left( 1 - 6 \left( \frac{2\pi}{8K} \right)^2 \kappa_2 \kappa_3 \kappa_4 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $S$  = Sierpiński's constant,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$g_+ = 5.58569469012941 \dots$$

$$g_+ = 5.5856946893(16)$$

$$g_+ = 5.5856946893(16)$$

prediction

CODATA 2022,  $\sigma = +0.52$

CODATA 2018,  $\sigma = +0.52$

$\Delta_{\text{precision}} = +0.00$

$\Delta_{\text{scaled}} = +0.00$

**neutron to shielded proton magnetic moment ratio**

1017.2(23)

$$\frac{\mu_n}{\mu_+} = -\frac{S}{C_{\text{CFP}}} \left( \frac{m_+}{m_n} \right) \left( 1 + \sqrt{5} \left( \frac{4\pi}{V_{\text{fe}}} \right) \mathfrak{K}_2 \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where  $S$  = Sierpiński's constant,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $m_+$  = the proton mass,  $m_n$  = the neutron mass,  $\pi$  = Archimedes' constant,  $V_{\text{fe}}$  = the figure eight knot hyperbolic volume,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_n/\mu_+' = -6.84997004903262 \dots \times 10^{-1}$	prediction
$\mu_n/\mu_+' = -6.8499694(16) \times 10^{-1}$	CODATA 2022, $\sigma = -0.41$
$\mu_n/\mu_+' = -6.8499694(16) \times 10^{-1}$	CODATA 2018, $\sigma = -0.41$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**electron-proton magnetic moment ratio**

974.3959(29)

$$\frac{\mu_e}{\mu_+} = -\frac{S}{P_{up}} \left( \frac{m_+}{m_e} \right) \left( 1 + \sqrt{7} \left( \frac{4x_\infty}{2K} \right) \mathfrak{K}_2 \mathfrak{K}_3 \mathfrak{K}_4 \boxtimes \right)$$

Where  $S$  = Sierpiński's constant,  $P_{up}$  = the universal parabolic constant,  $m_+$  = the proton mass,  $m_e$  = the electron mass,  $x_\infty$  = the 1<sup>st</sup> Foias constant,  $K$  = Catalan's constant,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_e/\mu_+ = -6.58210687490440 \dots \times 10^2$	prediction
$\mu_e/\mu_+ = -6.5821068789(19) \times 10^2$	CODATA 2022, $\sigma = +2.10$
$\mu_e/\mu_+ = -6.5821068789(20) \times 10^2$	CODATA 2018, $\sigma = -2.00$
	$\Delta_{\text{precision}} = +0.02$
	$\Delta_{\text{scaled}} = +0.00$

**electron to shielded proton magnetic moment ratio**

1231.146(41)

$$\frac{\mu_e}{\mu_+'} = -\frac{S}{P_{up}} \left( \frac{m_+}{m_e} \right) \left( 1 + \sqrt{5} \left( \frac{2K}{18} \right) \varkappa_2^2 \varkappa_3^2 \varkappa_4^2 \boxtimes \right)$$

Where  $S$  = Sierpiński's constant,  $P_{up}$  = the universal parabolic constant,  $m_+$  = the proton mass,  $m_e$  = the electron mass,  $K$  = Catalan's constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_e/\mu_+' = -6.58227587029051 \dots \times 10^2$	prediction
$\mu_e/\mu_+' = -6.582275856(27) \times 10^2$	CODATA 2022, $\sigma = -0.53$
$\mu_e/\mu_+' = -6.582275971(72) \times 10^2$	CODATA 2018, $\sigma = -1.40$
	$\Delta_{\text{precision}} = +0.42$
	$\Delta_{\text{scaled}} = -1.60$

**muon-proton magnetic moment ratio**

1036.95(23)

$$\frac{\mu_\mu}{\mu_+'} = -\frac{S}{P_{up}} \left( \frac{m_+}{m_\mu} \right) \left( 1 + \frac{\sqrt{35}}{2\pi} \left( \frac{4K}{18} \right) \varkappa_2^2 \varkappa_3^2 \varkappa_4^2 \boxtimes \right)$$

Where  $S$  = Sierpiński's constant,  $P_{up}$  = the universal parabolic constant,  $m_+$  = the proton mass,  $m_\mu$  = the muon mass,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_\mu/\mu_+' = -3.18334511221316 \dots$	prediction
$\mu_\mu/\mu_+' = -3.183345146(71)$	CODATA 2022, $\sigma = +0.48$
$\mu_\mu/\mu_+' = -3.183345142(71)$	CODATA 2018, $\sigma = -0.42$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.06$

## shielded proton magnetic moment to nuclear magneton ratio

-581.173(40)

$$\frac{\mu_+'}{\mu_N} = \frac{2 \varkappa_1 \varkappa_2^2}{S} \left( 1 - \frac{1}{e^{6\gamma}} \left( \frac{2\pi}{2K} \right) \varkappa_2^2 \varkappa_3^2 \varkappa_4^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $S$  = Sierpiński's constant,  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_+'/\mu_N = 2.79277562234849 \dots$	prediction
$\mu_+'/\mu_N = 2.792775648(11)$	CODATA 2022, $\sigma = -2.33$
$\mu_+'/\mu_N = 2.792775599(30)$	CODATA 2018, $\sigma = +0.78$
	$\Delta_{\text{precision}} = +0.44$
	$\Delta_{\text{scaled}} = +1.63$

## shielded proton magnetic moment to Bohr magneton ratio

-581.174(41)

$$\frac{\mu_+'}{\mu_B} = \frac{2 \varkappa_1 \varkappa_2^2}{S} \left( \frac{m_e}{m_+} \right) \left( 1 - \frac{1}{e^{6\gamma}} \left( \frac{2\pi}{2K} \right) \varkappa_2^2 \varkappa_3^2 \varkappa_4^2 \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $S$  = Sierpiński's constant,  $m_e$  = the electron mass,  $m_+$  = the proton mass,  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_+'/\mu_B = 1.52099314114035 \dots \times 10^{-3}$	prediction
$\mu_+'/\mu_B = 1.5209931551(62) \times 10^{-3}$	CODATA 2022, $\sigma = -2.25$
$\mu_+'/\mu_B = 1.520993128(17) \times 10^{-3}$	CODATA 2018, $\sigma = +0.78$
	$\Delta_{\text{precision}} = +1.44$
	$\Delta_{\text{scaled}} = +1.59$

**proton gyromagnetic ratio**

-324.098045(83)

$$\gamma_+ = \frac{2 \kappa_1^2 \kappa_2^2}{S} \left( \frac{q_p}{m_+} \right) \left( 1 - \frac{1}{5} \left( \frac{2\pi}{4s} \right) \kappa_2^2 \kappa_3^2 \kappa_4^2 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $S$  = Sierpiński's constant,  $q_p$  = the Planck charge,  $m_+$  = the proton mass,  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\gamma_+ = 2.67522187170411 \dots \times 10^8 \text{ 1/s} \cdot \text{T}$	prediction
$\gamma_+ = 2.6752218708(11) \times 10^8 \text{ 1/s} \cdot \text{T}$	CODATA 2018, $\sigma = +0.82$
$\gamma_+ = 2.6752218744(11) \times 10^8 \text{ 1/s} \cdot \text{T}$	CODATA 2018, $\sigma = -2.45$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = +3.27$

**proton gyromagnetic ratio in MHz/T**

-324.097998(83)

$$\dot{\gamma}_+ = \frac{\kappa_1^2 \kappa_2^2}{\pi S} \frac{1}{\text{MHz}} \left( \frac{s q_p}{m_+} \right) \left( 1 - \frac{1}{5} \left( \frac{2\pi}{4s} \right) \kappa_2^2 \kappa_3^2 \kappa_4^2 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $S$  = Sierpiński's constant, MHz = the megahertz, s = the second,  $q_p$  = the Planck charge,  $m_+$  = the proton mass,  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\dot{\gamma}_+ = 4.25774784749262 \dots \times 10^1 \text{ MHz/T}$	prediction
$\dot{\gamma}_+ = 4.2577478461(18) \times 10^1 \text{ MHz/T}$	CODATA 2022, $\sigma = +0.77$
$\dot{\gamma}_+ = 4.2577478518(18) \times 10^1 \text{ MHz/T}$	CODATA 2018, $\sigma = -2.39$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -3.17$

**proton magnetic moment**

-318.9479(42)

$$\mu_+ = \frac{\kappa_1^2 \kappa_2^2}{S} \left( \frac{l_p^2 q_p m_p}{t_p m_+} \right) \left( 1 - \frac{7}{5} \left( \frac{Im(\omega_1)}{6s} \right) \kappa_2^2 \kappa_3^2 \kappa_4^2 \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $S$  = Sierpiński's constant,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_+$  = the proton mass,  $\omega_1$  = the omega\_1 constant,  $s$  = the arc length of the unit lemniscate,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_+ = 1.41060679716311 \dots \times 10^{-26} \text{ J/T}$	prediction
$\mu_+ = 1.41060679545(60) \times 10^{-26} \text{ J/T}$	CODATA 2018, $\sigma = +2.86$
$\mu_+ = 1.41060679736(60) \times 10^{-26} \text{ J/T}$	CODATA 2018, $\sigma = -0.32$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -3.18$

**Faraday constant**

-283.17057(51)

$$F = \frac{6}{e^\gamma} \kappa_1^3 \left( \frac{1}{m_p} \right) \left( 1 - \left( \frac{2K}{35} \right) \kappa_2^2 \kappa_3^2 \kappa_4^2 \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $m_p$  = the Planck mass,  $K$  = Catalan's constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$F = 9.64853322042857 \dots \times 10^4$ C/mol	prediction
$F = 9.648533212 \times 10^4$ C/mol	CODATA 2022, 9.06-digit match
$F = 9.648533212 \times 10^4$ C/mol	CODATA 2018, 9.06-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**molar Planck constant**

-278.3994(13)

$$M_p = \frac{6}{e^\gamma} \kappa_1^2 2\pi \left( \frac{l_p^2}{s^2 t_p q_p} \right) \left( 1 - \frac{5}{\sqrt{32}} \left( \frac{2K}{6s} \right) \kappa_2^2 \kappa_3^2 \kappa_4^2 \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\pi$  = Archimedes' constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $s$  = the second,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $K$  = Catalan's constant,  $s$  = the arc length of the unit lemniscate,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$M_p = 3.99031271062295 \dots \times 10^{-10}$ J/Hz mol	prediction
$M_p = 3.990312712 \times 10^{-10}$ J/Hz mol	CODATA 2022, 9.46-digit match
$M_p = 3.990312712 \times 10^{-10}$ J/Hz mol	CODATA 2018, 9.46-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**molar volume of silicon**

3747.90(50)

$$V_m(Si) = \frac{6}{e^\gamma} \left( \frac{2\pi}{32s} \right)^3 \frac{1}{\kappa_1^{10}} \left( \frac{l_p^3 m_p^2}{q_p m_e^3} \right) \left( 1 + \frac{32}{18^2} \left( \frac{4\pi}{D_d} \right) \kappa_2^2 \kappa_3^2 \kappa_4^2 \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\pi$  = Archimedes' constant,  $s$  = the arc length of the unit lemniscate,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $q_p$  = the Planck charge,  $m_e$  = the electron mass,  $D_d$  = the dimer constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$V_m(Si) = 1.20588317215772 \dots \times 10^{-5} \text{ m}^3/\text{mol} \quad \text{prediction}$$

$$V_m(Si) = 1.205883199(60) \times 10^{-5} \text{ m}^3/\text{mol} \quad \text{CODATA 2018, } \sigma = -0.45$$

$$V_m(Si) = 1.205883199(60) \times 10^{-5} \text{ m}^3/\text{mol} \quad \text{CODATA 2018, } \sigma = -0.45$$

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.00$$

## speed of light in vacuum

$$c = \left( \frac{l_p}{t_p} \right) \left( 1 + \sqrt{G_{Gi}} (\kappa_1 - \kappa_2) \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $t_p$  = the Planck time,  $G_{Gi}$  = Gieseeking's constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$c = 2.99792458097898 \dots \times 10^8 \text{ m/s}$	prediction
$c = 2.99792458 \times 10^8 \text{ m/s}$	CODATA 2022, 9-digit match
$c = 2.99792458 \times 10^8 \text{ m/s}$	CODATA 2018, 9-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

Also listed as the *natural unit of velocity*.

## inverse meter-hertz relationship

-3.595(16)

$$\frac{1}{\text{m}} : \text{Hz} = \left( \frac{l_p}{t_p \text{ m}} \right) \left( 1 + \sqrt{G_{Gi}} (\kappa_1 - \kappa_2) \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $t_p$  = the Planck time, m = the meter,  $G_{Gi}$  = Gieseeking's constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$1/\text{m} : \text{Hz} = 2.99792458097898 \dots \times 10^8 \text{ Hz}$	prediction
$1/\text{m} : \text{Hz} = 2.99792458 \times 10^8 \text{ Hz}$	CODATA 2022, 9-digit match
$1/\text{m} : \text{Hz} = 2.99792458 \times 10^8 \text{ Hz}$	CODATA 2018, 9-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**classical electron radius**

8.7468(46)

$$r_e = \kappa_1^2 \left( \frac{l_p m_p}{m_e} \right) \left( 1 - \sqrt{6 C_{R1}} (\kappa_1 - \kappa_2) \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $C_{R1}$  = Ramanujan's 1<sup>st</sup> continued fraction constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$r_e = 2.81794032699479 \dots \times 10^{-15} \text{ m}$	prediction
$r_e = 2.8179403205(13) \times 10^{-15} \text{ m}$	CODATA 2022, $\sigma = +5.00$
$r_e = 2.8179403262(13) \times 10^{-15} \text{ m}$	CODATA 2018, $\sigma = +0.61$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -4.38$

**atomic unit of length**

8.7701(15)

$$a_0 = \frac{1}{\kappa_1^2} \left( \frac{l_p m_p}{m_e} \right) \left( 1 - \sqrt{6 C_{R1}} (\kappa_1 - \kappa_2) \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $C_{R1}$  = Ramanujan's 1<sup>st</sup> continued fraction constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$a_0 = 5.29177210521101 \dots \times 10^{-11} \text{ m}$	prediction
$a_0 = 5.29177210544(82) \times 10^{-11} \text{ m}$	CODATA 2022, $\sigma = -0.28$
$a_0 = 5.29177210903(80) \times 10^{-11} \text{ m}$	CODATA 2018, $\sigma = -4.77$
	$\Delta_{\text{precision}} = -0.03$
	$\Delta_{\text{scaled}} = -4.48$

Also listed as the *Bohr radius*.

## Thomson cross section

17.5080(90)

$$\sigma_e = \sqrt{\frac{32}{18}} 2\pi \kappa_1^4 \left(\frac{l_p m_p}{m_e}\right)^2 \left(1 - 2\sqrt{6 C_{R1}} (\kappa_1 - \kappa_2) \boxtimes\right)$$

Where  $\pi$  = Archimedes' constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $C_{R1}$  = Ramanujan's 1<sup>st</sup> continued fraction constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \sigma_e &= 6.65245873589771 \dots \times 10^{-29} \text{ m}^2 && \text{prediction} \\ \sigma_e &= 6.6524587051(62) \times 10^{-29} \text{ m}^2 && \text{CODATA 2022, } \sigma = +4.97 \\ \sigma_e &= 6.6524587321(60) \times 10^{-29} \text{ m}^2 && \text{CODATA 2018, } \sigma = +0.63 \\ &&& \Delta_{\text{precision}} = -0.03 \\ &&& \Delta_{\text{scaled}} = -4.50 \end{aligned}$$

## Fermi coupling constant

-243.6(51)

$$\frac{G_F}{(\hbar c)^3} = \sqrt{\frac{32}{18}} \left(2\pi \frac{\text{joule}}{\text{GeV}}\right)^2 \left(\frac{t_p^4}{l_p^4 m_p \text{ kg}}\right) \left(1 + 28\sqrt{6 C_{R1}} (\kappa_1 - \kappa_2) \boxtimes\right)$$

Where  $\pi$  = Archimedes' constant, GeV = the gigaelectron volt,  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $m_p$  = the Planck mass, kg = the kilogram,  $C_{R1}$  = Ramanujan's 1<sup>st</sup> continued fraction constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} G_F/(\hbar c)^3 &= 1.16637852183353 \dots \times 10^{-5} \text{ GeV}^{-2} && \text{prediction} \\ G_F/(\hbar c)^3 &= 1.1663787(06) \times 10^{-5} \text{ GeV}^{-2} && \text{CODATA 2022, } \sigma = -0.30 \\ G_F/(\hbar c)^3 &= 1.1663787(06) \times 10^{-5} \text{ GeV}^{-2} && \text{CODATA 2018, } \sigma = -0.30 \\ &&& \Delta_{\text{precision}} = +0.00 \\ &&& \Delta_{\text{scaled}} = +0.00 \end{aligned}$$

**atomic mass constant**

-1.3664(31)

$$A_{\text{mass}} = \sqrt{\frac{18}{32} \frac{P_{up}}{\varkappa_1^6}} (m_e) \left( 1 + \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $P_{up}$  = the universal parabolic constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{mass}} = 1.66053906821322 \dots \times 10^{-27} \text{ kg}$	prediction
$A_{\text{mass}} = 1.66053906892(52) \times 10^{-27} \text{ kg}$	CODATA 2022, $\sigma = -1.36$
$A_{\text{mass}} = 1.66053906660(50) \times 10^{-27} \text{ kg}$	CODATA 2018, $\sigma = +3.23$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +4.64$

Also listed as the *atomic mass unit-kilogram relationship*, and the *unified atomic mass unit*.

**kilogram-atomic mass unit relationship**

1.3665(30)

$$\text{kg} : A_{\text{mass}} = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{\text{kg}}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant, kg = the kilogram,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\text{kg} : A_{\text{mass}} = 6.02214075623046 \dots \times 10^{26} \text{ u}$	prediction
$\text{kg} : A_{\text{mass}} = 6.0221407537(19) \times 10^{26} \text{ u}$	CODATA 2022, $\sigma = +1.33$
$\text{kg} : A_{\text{mass}} = 6.0221407621(18) \times 10^{26} \text{ u}$	CODATA 2018, $\sigma = -3.26$
	$\Delta_{\text{precision}} = -0.03$
	$\Delta_{\text{scaled}} = -4.66$

**electron relative atomic mass**

1.36644(29)

$$A_r(e) = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_r(e) = 5.48579909049541 \dots \times 10^{-4}$	prediction
$A_r(e) = 5.485799090441(97) \times 10^{-4}$	CODATA 2022, $\sigma = +0.56$
$A_r(e) = 5.48579909065(16) \times 10^{-4}$	CODATA 2018, $\sigma = -0.97$
	$\Delta_{\text{precision}} = -0.78$
	$\Delta_{\text{scaled}} = 1.31$

**proton relative atomic mass**

1.36639(53)

$$A_r(+)= \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{m_+}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_+$  = the proton mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_r(+)= 1.00727646658226 \dots$	prediction
$A_r(+)= 1.0072764665789(83)$	CODATA 2022, $\sigma = +0.41$
$A_r(+)= 1.007276466621(53)$	CODATA 2018, $\sigma = -0.73$
	$\Delta_{\text{precision}} = -0.20$
	$\Delta_{\text{scaled}} = -0.81$

**neutron relative atomic mass**

1.3653(48)

$$A_r(n) = \sqrt{\frac{32 \kappa_1^6}{18 P_{up}} \left( \frac{m_n}{m_e} \right) \left( 1 - \frac{1}{L} (\kappa_1 - \kappa_2) \boxtimes \right)}$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_n$  = the neutron mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_r(n) = 1.00866491547289 \dots$	prediction
$A_r(n) = 1.00866491606(40)$	CODATA 2022, $\sigma = -1.47$
$A_r(n) = 1.00866491595(49)$	CODATA 2018, $\sigma = -0.97$
	$\Delta_{\text{precision}} = +0.09$
	$\Delta_{\text{scaled}} = +0.22$

**deuteron relative atomic mass**

1.36674(20)

$$A_r(\text{de}) = \sqrt{\frac{32 \kappa_1^6}{18 P_{up}} \left( \frac{m_{\text{de}}}{m_e} \right) \left( 1 - \frac{1}{L} (\kappa_1 - \kappa_2) \boxtimes \right)}$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_{\text{de}}$  = the deuteron mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_r(\text{de}) = 2.01355321254847 \dots$	prediction
$A_r(\text{de}) = 2.013553212544(15)$	CODATA 2022, $\sigma = +0.32$
$A_r(\text{de}) = 2.013553212745(40)$	CODATA 2018, $\sigma = -4.91$
	$\Delta_{\text{precision}} = +0.43$
	$\Delta_{\text{scaled}} = -5.03$

**helion relative atomic mass**

1.36637(32)

$$A_r(\text{he}) = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{m_{\text{he}}}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_{\text{he}}$  = the helion mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_r(\text{he}) = 3.01493224693074 \dots$	prediction
$A_r(\text{he}) = 3.014932246932(74)$	CODATA 2022, $\sigma = -0.02$
$A_r(\text{he}) = 3.014932247175(97)$	CODATA 2018, $\sigma = -2.52$
	$\Delta_{\text{precision}} = +0.12$
	$\Delta_{\text{scaled}} = -2.51$

**triton relative atomic mass**

1.36615(40)

$$A_r(\text{tri}) = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{m_{\text{tri}}}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_{\text{tri}}$  = the triton mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_r(\text{tri}) = 3.01550071572160 \dots$	prediction
$A_r(\text{tri}) = 3.01550071597(10)$	CODATA 2022, $\sigma = -2.48$
$A_r(\text{tri}) = 3.01550071621(12)$	CODATA 2018, $\sigma = -4.07$
	$\Delta_{\text{precision}} = +0.08$
	$\Delta_{\text{scaled}} = -2.00$

**alpha particle relative atomic mass**

1.36577(15)

$$A_r(\alpha) = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{m_\alpha}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_\alpha$  = the alpha particle mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_r(\alpha) = 4.00150617912918 \dots$	prediction
$A_r(\alpha) = 4.001506179129(62)$	CODATA 2022, $\sigma = +0.00$
$A_r(\alpha) = 4.001506179127(63)$	CODATA 2018, $\sigma = +0.03$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.03$

**electron mass in u**

1.36644(29)

$$\frac{m_e}{A_{\text{mass}}} = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $A_{\text{mass}}$  = atomic mass constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_e/A_{\text{mass}} = 5.48579909049541 \dots \times 10^{-4} \text{ u}$	prediction
$m_e/A_{\text{mass}} = 5.485799090441(97) \times 10^{-4} \text{ u}$	CODATA 2022, $\sigma = +0.56$
$m_e/A_{\text{mass}} = 5.48579909065(16) \times 10^{-4} \text{ u}$	CODATA 2018, $\sigma = -0.96$
	$\Delta_{\text{precision}} = +0.22$
	$\Delta_{\text{scaled}} = -1.25$

**neutron-proton mass difference in u**

0.63(3.52)

$$\frac{m_{\Delta}}{A_{\text{mass}}} = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{m_n - m_+}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_n$  = the neutron mass,  $m_+$  = the proton mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_{\Delta}/A_{\text{mass}} = 1.38844889062885 \dots \times 10^{-3} \text{ u}$	prediction
$m_{\Delta}/A_{\text{mass}} = 1.38844948(40) \times 10^{-3} \text{ u}$	CODATA 2022, $\sigma = -1.40$
$m_{\Delta}/A_{\text{mass}} = 1.38844933(49) \times 10^{-3} \text{ u}$	CODATA 2018, $\sigma = -0.90$
	$\Delta_{\text{precision}} = +0.09$
	$\Delta_{\text{scaled}} = +0.31$

**muon mass in u**

1.40(22)

$$\frac{m_{\mu}}{A_{\text{mass}}} = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{m_{\mu}}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_{\mu}$  = the muon mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_{\mu}/A_{\text{mass}} = 1.13428925044824 \dots \times 10^{-1} \text{ u}$	prediction
$m_{\mu}/A_{\text{mass}} = 1.134289257(25) \times 10^{-1} \text{ u}$	CODATA 2022, $\sigma = -0.26$
$m_{\mu}/A_{\text{mass}} = 1.134289259(25) \times 10^{-1} \text{ u}$	CODATA 2018, $\sigma = -0.34$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -0.08$

**proton mass in u**

1.36639(53)

$$\frac{m_+}{A_{\text{mass}}} = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{m_+}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_+$  = the proton mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_+/A_{\text{mass}} = 1.00727646658226 \dots \text{ u}$	prediction
$m_+/A_{\text{mass}} = 1.0072764665789(83) \text{ u}$	CODATA 2022, $\sigma = +0.41$
$m_+/A_{\text{mass}} = 1.007276466621(53) \text{ u}$	CODATA 2018, $\sigma = -0.73$
	$\Delta_{\text{precision}} = +0.81$
	$\Delta_{\text{scaled}} = -2.06$

**neutron mass in u**

1.3653(48)

$$\frac{m_n}{A_{\text{mass}}} = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{m_n}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_n$  = the neutron mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_n/A_{\text{mass}} = 1.00866491547289 \dots \text{ u}$	prediction
$m_n/A_{\text{mass}} = 1.00866491606(40) \text{ u}$	CODATA 2022, $\sigma = -1.47$
$m_n/A_{\text{mass}} = 1.00866491595(49) \text{ u}$	CODATA 2018, $\sigma = -0.97$
	$\Delta_{\text{precision}} = +0.09$
	$\Delta_{\text{scaled}} = +0.22$

**tau mass in u**

-3.0(678)

$$\frac{m_\tau}{A_{\text{mass}}} = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{m_\tau}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_\tau$  = the tau mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_\tau/A_{\text{mass}} = 1.90754046521642 \dots \text{ u}$	prediction
$m_\tau/A_{\text{mass}} = 1.90754(13) \text{ u}$	CODATA 2022, $\sigma = +0.00$
$m_\tau/A_{\text{mass}} = 1.90754(13) \text{ u}$	CODATA 2018, $\sigma = +0.00$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**deuteron mass in u**

1.36674(20)

$$\frac{m_{\text{de}}}{A_{\text{mass}}} = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{m_{\text{de}}}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_{\text{de}}$  = the deuteron mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_{\text{de}}/A_{\text{mass}} = 2.01355321254878 \dots \text{ u}$	prediction
$m_{\text{de}}/A_{\text{mass}} = 2.013553212544(15) \text{ u}$	CODATA 2022, $\sigma = +0.32$
$m_{\text{de}}/A_{\text{mass}} = 2.013553212745(40) \text{ u}$	CODATA 2018, $\sigma = -4.91$
	$\Delta_{\text{precision}} = +0.43$
	$\Delta_{\text{scaled}} = -5.03$

**helion mass in u**

1.36637(32)

$$\frac{m_{\text{he}}}{A_{\text{mass}}} = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{m_{\text{he}}}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_{\text{he}}$  = the helion mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_{\text{he}}/A_{\text{mass}} = 3.01493224693074 \dots \text{ u}$	prediction
$m_{\text{he}}/A_{\text{mass}} = 3.014932246932(74) \text{ u}$	CODATA 2022, $\sigma = -0.02$
$m_{\text{he}}/A_{\text{mass}} = 3.014932247175(97) \text{ u}$	CODATA 2018, $\sigma = -2.52$
	$\Delta_{\text{precision}} = +0.12$
	$\Delta_{\text{scaled}} = -2.51$

**triton mass in u**

1.36715(40)

$$\frac{m_{\text{tri}}}{A_{\text{mass}}} = \sqrt{\frac{32}{18} \frac{\varkappa_1^6}{P_{up}}} \left( \frac{m_{\text{tri}}}{m_e} \right) \left( 1 - \frac{1}{L} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_{\text{tri}}$  = the triton mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_{\text{tri}}/A_{\text{mass}} = 3.01550071572160 \dots \text{ u}$	prediction
$m_{\text{tri}}/A_{\text{mass}} = 3.01550071597(10) \text{ u}$	CODATA 2022, $\sigma = -2.48$
$m_{\text{tri}}/A_{\text{mass}} = 3.01550071621(12) \text{ u}$	CODATA 2018, $\sigma = -4.15$
	$\Delta_{\text{precision}} = +0.08$
	$\Delta_{\text{scaled}} = -2.00$

## alpha particle mass in u

1.36577(15)

$$\frac{m_\alpha}{A_{\text{mass}}} = \sqrt{\frac{32 \kappa_1^6}{18 P_{up}}} \left( \frac{m_\alpha}{m_e} \right) \left( 1 - \frac{1}{L} (\kappa_1 - \kappa_2) \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_\alpha$  = the alpha particle mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$m_\alpha/A_{\text{mass}} = 4.00150617912918 \dots \text{ u}$$

$$m_\alpha/A_{\text{mass}} = 4.001506179129(62) \text{ u}$$

$$m_\alpha/A_{\text{mass}} = 4.001506179127(63) \text{ u}$$

prediction

CODATA 2022,  $\sigma = +0.00$

CODATA 2018,  $\sigma = +0.03$

$\Delta_{\text{precision}} = +0.00$

$\Delta_{\text{scaled}} = +0.03$

**electron volt-atomic mass unit relationship**

8.9646(30)

$$eV : A_{\text{mass}} = \text{joule}^2 \sqrt{\frac{32 \kappa_1^8}{18 P_{up}} \left( \frac{t_p^2 q_p}{l_p^2 C m_e} \right)} \left( 1 - \alpha_F (\kappa_1 - \kappa_2) \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $l_p$  = the Planck length,  $C$  = the coulomb,  $m_e$  = the electron mass,  $\alpha_F$  = the alpha Feigenbaum constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

eV : $A_{\text{mass}} = 1.07354410142667 \dots \times 10^{-9} \text{ u}$	prediction
eV : $A_{\text{mass}} = 1.07354410083(33) \times 10^{-9} \text{ u}$	CODATA 2022, $\sigma = +1.75$
eV : $A_{\text{mass}} = 1.07354410233(32) \times 10^{-9} \text{ u}$	CODATA 2018, $\sigma = -2.82$
	$\Delta_{\text{precision}} = -0.03$
	$\Delta_{\text{scaled}} = -4.68$

**atomic mass unit-electron volt relationship**

-8.9646(30)

$$A_{\text{mass}} : eV = \sqrt{\frac{18 P_{up}}{32 \kappa_1^8} \left( \frac{l_p^2 C m_e}{t_p^2 q_p} \right)} \left( 1 + \alpha_F (\kappa_1 - \kappa_2) \boxtimes \right)$$

Where  $P_{up}$  = the universal parabolic constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $C$  = the coulomb,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $\alpha_F$  = the alpha Feigenbaum constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

A <sub>mass</sub> : eV = 9.31494103195445 ... × 10 <sup>8</sup> eV	prediction
A <sub>mass</sub> : eV = 9.3149410372(29) × 10 <sup>8</sup> eV	CODATA 2022, $\sigma = -1.81$
A <sub>mass</sub> : eV = 9.3149410242(28) × 10 <sup>8</sup> eV	CODATA 2018, $\sigma = +2.77$
	$\Delta_{\text{precision}} = -0.04$
	$\Delta_{\text{scaled}} = +4.64$

**joule-inverse meter relationship**

-1.5349(10)

$$J : \frac{1}{m} = \frac{\text{joule}}{2\pi} \left( \frac{t_p^2}{l_p^3 m_p} \right) \left( 1 + \frac{2}{\delta_F} (\kappa_1 - \kappa_2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $\delta_F$  = the delta Feigenbaum constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} J : 1/m &= 5.03411656778384 \dots \times 10^{24} \text{ cycles/m} && \text{prediction} \\ J : 1/m &= 5.034116567 \times 10^{24} \text{ cycles/m} && \text{CODATA 2022, 10-digit match} \\ J : 1/m &= 5.034116567 \times 10^{24} \text{ cycles/m} && \text{CODATA 2018, 10-digit match} \\ &&& \Delta_{\text{precision}} = +0.00 \\ &&& \Delta_{\text{scaled}} = +0.00 \end{aligned}$$

**electron volt-inverse meter relationship**

-1.53673(62)

$$\text{eV} : \frac{1}{m} = \frac{\text{eV}}{2\pi} \left( \frac{t_p^2}{l_p^3 m_p} \right) \left( 1 + \frac{2}{\delta_F} (\kappa_1 - \kappa_2) \boxtimes \right)$$

Where eV = the electron-volt,  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $\delta_F$  = the delta Feigenbaum constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \text{eV} : 1/m &= 8.06554393884387 \dots \times 10^5 \text{ cycles/m} && \text{prediction} \\ \text{eV} : 1/m &= 8.065543937 \times 10^5 \text{ cycles/m} && \text{CODATA 2022, 9.64-digit match} \\ \text{eV} : 1/m &= 8.065543937 \times 10^5 \text{ cycles/m} && \text{CODATA 2018, 9.64-digit match} \\ &&& \Delta_{\text{precision}} = +0.00 \\ &&& \Delta_{\text{scaled}} = +0.00 \end{aligned}$$

**inverse meter-electron volt relationship**

1.5376(41)

$$\frac{1}{\text{m}} : \text{eV} = 2\pi \frac{\text{joule}}{\text{eV}} \left( \frac{l_p^3 m_p}{t_p^2 \text{m}} \right) \left( 1 - \frac{2}{\delta_F} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant, eV = the electron-volt,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time, m = the meter,  $\delta_F$  = the delta Feigenbaum constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

1/m : eV = 1.239841984 <b>10221</b> ... × 10 <sup>-6</sup> eV	prediction
1/m : eV = 1.239841984 × 10 <sup>-6</sup> eV	CODATA 2022, 10-digit match
1/m : eV = 1.239841984 × 10 <sup>-6</sup> eV	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**inverse meter-joule relationship**

1.5349(10)

$$\frac{1}{\text{m}} : \text{J} = 2\pi \left( \frac{l_p^3 m_p}{t_p^2 \text{m}} \right) \left( 1 - \frac{2}{\delta_F} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time, m = the meter,  $\delta_F$  = the delta Feigenbaum constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

1/m : J = 1.986445857 <b>05373</b> ... × 10 <sup>-25</sup> J	prediction
1/m : J = 1.986445857 × 10 <sup>-25</sup> J	CODATA 2022, 10-digit match
1/m : J = 1.986445857 × 10 <sup>-25</sup> J	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**reduced Planck constant times c in MeV fm**

1.5345(13)

$$\hbar c = \frac{\text{joule}}{\text{MeV fm}} \left( \frac{l_p^3 \text{ m } m_p}{t_p^2} \right) \left( 1 - \frac{2}{\delta_F} (\varkappa_1 - \varkappa_2) \boxtimes \right)$$

Where MeV = the megaelectron-volt, fm = the femtometer,  $l_p$  = the Planck length, m = the meter,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $\delta_F$  = the delta Feigenbaum constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\hbar c = 1.97326980422731 \dots \times 10^2 \text{ MeV fm}$	prediction
$\hbar c = 1.973269804 \times 10^2 \text{ MeV fm}$	CODATA 2022, 10-digit match
$\hbar c = 1.973269804 \times 10^2 \text{ MeV fm}$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**atomic unit of electric potential**

-6.251583(11)

$$A_{e \text{ pot}} = \sqrt{\frac{32 \kappa_1^{12}}{18 P_{up}} \left( \frac{l_p^2 A_{\text{mass}}}{t_p^2 q_p} \right)} \left( 1 - \frac{35}{18} \left( \frac{4}{\delta_F} \right) (\kappa_1 + \kappa_2) \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $l_p$  = the Planck length,  $A_{\text{mass}}$  = the atomic mass constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $\delta_F$  = the delta Feigenbaum constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{e \text{ pot}} = 2.72113862459609 \dots \times 10^1 \text{ V}$	prediction
$A_{e \text{ pot}} = 2.7211386245981(30) \times 10^1 \text{ V}$	CODATA 2022, $\sigma = -0.67$
$A_{e \text{ pot}} = 2.7211386245988(53) \times 10^1 \text{ V}$	CODATA 2018, $\sigma = -0.51$
	$\Delta_{\text{precision}} = +0.25$
	$\Delta_{\text{scaled}} = -0.13$

**Rydberg constant times hc in eV**

-6.251583(11)

$$R_{\infty} hc = \frac{1}{2} \sqrt{\frac{32 \kappa_1^{12}}{18 P_{up}} \left( \frac{l_p^2 C A_{\text{mass}}}{t_p^2 q_p} \right)} \left( 1 - \frac{35}{18} \left( \frac{4}{\delta_F} \right) (\kappa_1 + \kappa_2) \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $l_p$  = the Planck length,  $C$  = the coulomb,  $A_{\text{mass}}$  = the atomic mass constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $\delta_F$  = the delta Feigenbaum constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$R_{\infty} hc = 1.36056931229804 \dots \times 10^1 \text{ eV}$	prediction
$R_{\infty} hc = 1.3605693122990(15) \times 10^1 \text{ eV}$	CODATA 2022, $\sigma = -0.64$
$R_{\infty} hc = 1.3605693122994(26) \times 10^1 \text{ eV}$	CODATA 2018, $\sigma = -0.52$
	$\Delta_{\text{precision}} = +0.24$
	$\Delta_{\text{scaled}} = -0.15$

**hartree-electron volt relationship**

-6.251583(11)

$$E_h : eV = \sqrt{\frac{32}{18} \frac{\varkappa_1^{12}}{P_{up}}} \left( \frac{l_p^2 C A_{mass}}{t_p^2 q_p} \right) \left( 1 - \frac{35}{18} \left( \frac{4}{\delta_F} \right) (\varkappa_1 + \varkappa_2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $l_p$  = the Planck length,  $C$  = the coulomb,  $A_{mass}$  = the atomic mass constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $\delta_F$  = the delta Feigenbaum constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_h : eV = 2.72113862459609 \dots \times 10^1 eV$	prediction
$E_h : eV = 2.7211386245981(30) \times 10^1 eV$	CODATA 2022, $\sigma = -0.67$
$E_h : eV = 2.7211386245988(53) \times 10^1 eV$	CODATA 2018, $\sigma = -0.51$
	$\Delta_{precision} = +0.25$
	$\Delta_{scaled} = -0.13$

Also listed as the *Hartree energy in eV*.

**electron volt-hartree relationship**

6.251583(11)

$$eV : E_h = \sqrt{\frac{18}{32} \frac{P_{up}}{\varkappa_1^{12}}} \left( \frac{t_p^2 q_p}{l_p^2 C A_{mass}} \right) \left( 1 + \frac{35}{18} \left( \frac{4}{\delta_F} \right) (\varkappa_1 + \varkappa_2) \boxtimes \right)$$

Where  $P_{up}$  = the universal parabolic constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $l_p$  = the Planck length,  $C$  = the coulomb,  $A_{mass}$  = the atomic mass constant,  $\delta_F$  = the delta Feigenbaum constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$eV : E_h = 3.67493221756773 \dots \times 10^{-2} E_h$	prediction
$eV : E_h = 3.6749322175665(40) \times 10^{-2} E_h$	CODATA 2022, $\sigma = +0.31$
$eV : E_h = 3.6749322175655(71) \times 10^{-2} E_h$	CODATA 2018, $\sigma = +0.31$
	$\Delta_{precision} = +0.25$
	$\Delta_{scaled} = +0.14$

**conventional value of farad-90**

-0.177949(50)

$$F_{90} = \text{farad} \left( 1 + \frac{C_d}{6} (\kappa_1 + \kappa_3 + \kappa_4) \boxtimes \right)$$

Where  $C_d$  = the domino tiling constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$F_{90} = 0.999999982178026 \dots$	F	prediction
$F_{90} = 0.99999998220$	F	CODATA 2022, 9.66-digit match
$F_{90} = 0.99999998220$	F	CODATA 2018, 9.66-digit match
		$\Delta_{\text{precision}} = +0.00$
		$\Delta_{\text{scaled}} = +0.00$

**conventional value of henry-90**

0.177949(50)

$$H_{90} = \text{henry} \left( 1 - \frac{C_d}{6} (\kappa_1 + \kappa_3 + \kappa_4) \boxtimes \right)$$

Where  $C_d$  = the domino tiling constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$H_{90} = 1.00000001782197 \dots$	H	prediction
$H_{90} = 1.00000001779$	H	CODATA 2022, 10.50-digit match
$H_{90} = 1.00000001779$	H	CODATA 2018, 10.50-digit match
		$\Delta_{\text{precision}} = +0.00$
		$\Delta_{\text{scaled}} = +0.00$

**conventional value of ohm-90**

0.177949(50)

$$\Omega_{90} = \text{ohm} \left( 1 - \frac{C_d}{6} (\kappa_1 + \kappa_3 + \kappa_4) \boxtimes \right)$$

Where  $C_d$  = the domino tiling constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\Omega_{90} = 1.00000001782197 \dots \Omega$	prediction
$\Omega_{90} = 1.00000001779 \Omega$	CODATA 2022, 10.50-digit match
$\Omega_{90} = 1.00000001779 \Omega$	CODATA 2018, 10.50-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**neutron-proton mass difference**

-7.3(29)

$$m_{\Delta} = (m_n - m_+) \left( 1 - 35 \frac{C_d}{6} (\kappa_1 + \kappa_3 + \kappa_4) \boxtimes \right)$$

Where  $m_n$  = the neutron mass,  $m_+$  = the proton mass,  $C_d$  = the domino tiling constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constants,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$m_{\Delta} = 2.30557506525214 \dots \times 10^{-30} \text{ kg}$	prediction
$m_{\Delta} = 2.30557461(67) \times 10^{-30} \text{ kg}$	CODATA 2022, $\sigma = +0.68$
$m_{\Delta} = 2.30557435(82) \times 10^{-30} \text{ kg}$	CODATA 2018, $\sigma = -2.86$
	$\Delta_{\text{precision}} = +0.09$
	$\Delta_{\text{scaled}} = +0.32$

**helion shielding shift**

-599.67077(23)

$$\sigma_{\text{he}} = 1 - \left( 1 - \sqrt{\frac{7 s^4}{\mathcal{P}^8 (2\pi)^3} (\mathfrak{K}_1 - \mathfrak{K}_3 - \mathfrak{K}_4) \boxtimes} \right)$$

Where  $\mathcal{P}$  = the prime constant,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $\mathfrak{K}_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\sigma_{\text{he}} = 5.99670299153336 \dots \times 10^{-5}$	prediction
$\sigma_{\text{he}} = 5.9967029(23) \times 10^{-5}$	CODATA 2022, $\sigma = +0.04$
$\sigma_{\text{he}} = 5.996743(10) \times 10^{-5}$	CODATA 2018, $\sigma = -4.00$
	$\Delta_{\text{precision}} = +0.64$
	$\Delta_{\text{scaled}} = -4.10$

**shielding difference of t and p in HT**

-0.239450(20)

$$\sigma_{\text{tp}} = 1 - \left( 1 - \frac{1}{7} \sqrt{\frac{s^4}{2^4 (2\pi)^3} (\mathfrak{K}_1 - \mathfrak{K}_3 - \mathfrak{K}_4) \boxtimes} \right)$$

Where  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $\mathfrak{K}_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\sigma_{\text{tp}} = 2.39369251464032 \dots \times 10^{-8}$	prediction
$\sigma_{\text{tp}} = 2.39450(20) \times 10^{-8}$	CODATA 2022, $\sigma = -4.04$
$\sigma_{\text{tp}} = 2.4140(20) \times 10^{-8}$	CODATA 2018, $\sigma = -10.15$
	$\Delta_{\text{precision}} = +1.00$
	$\Delta_{\text{scaled}} = -9.75$

**shielding difference of d and p in HD**

-0.198770(10)

$$\sigma_{dp} = 1 - \left( 1 - \frac{1}{3} \sqrt{\frac{5 s^4}{2^4 (2\pi)^5}} (\kappa_1 - \kappa_3 - \kappa_4) \boxtimes \right)$$

Where  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\sigma_{dp} = 1.98769745110889 \dots \times 10^{-8}$	prediction
$\sigma_{dp} = 1.98770(10) \times 10^{-8}$	CODATA 2022, $\sigma = -0.03$
$\sigma_{dp} = 2.0200(20) \times 10^{-8}$	CODATA 2018, $\sigma = -16.15$
	$\Delta_{\text{precision}} = +1.30$
	$\Delta_{\text{scaled}} = -16.15$

**proton magnetic shielding correction**

-256.715(41)

$$\sigma_+' = 1 - \left( 1 - 35 \sqrt{\frac{s^8}{2^4 (2\pi)^5}} (\kappa_1 - \kappa_3 - \kappa_4) \boxtimes \right)$$

Where  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\sigma_+' = 2.56684124759787 \dots \times 10^{-5}$	prediction
$\sigma_+' = 2.56715(41) \times 10^{-5}$	CODATA 2022, $\sigma = -0.75$
$\sigma_+' = 2.5689(11) \times 10^{-5}$	CODATA 2018, $\sigma = -1.87$
	$\Delta_{\text{precision}} = +0.43$
	$\Delta_{\text{scaled}} = -1.64$

**atomic unit of electric polarizability**

25.5252(31)

$$A_{ep} = \frac{1}{\varkappa_1^6} \left( \frac{t_p^2 q_p^2 m_p^2}{m_e^3} \right) \left( 1 - 18 \sqrt{\frac{s^9}{2^{11} (2\pi)^5}} (\varkappa_1 + \varkappa_3 + \varkappa_4) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $L$  = the lemniscate constant,  $\pi$  = Archimedes' constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{ep} = 1.64877727300424 \dots \times 10^{-41} \text{ m}^2 \cdot \text{C}^2/\text{J}$	prediction
$A_{ep} = 1.64877727212(51) \times 10^{-41} \text{ m}^2 \cdot \text{C}^2/\text{J}$	CODATA 2018, $\sigma = +1.73$
$A_{ep} = 1.64877727436(50) \times 10^{-41} \text{ m}^2 \cdot \text{C}^2/\text{J}$	CODATA 2018, $\sigma = -2.71$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = -4.48$

**atomic unit of electric field**

-16.3913(16)

$$A_{ef} = \varkappa_1^5 \left( \frac{l_p m_e^2}{t_p^2 q_p m_p} \right) \left( 1 + 18 \sqrt{\frac{s^8}{24 (2\pi)^7}} (\varkappa_1 + \varkappa_3 + \varkappa_4) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{ef} = 5.14220674774742 \dots \times 10^{11} \text{ V/m}$	prediction
$A_{ef} = 5.14220675112(80) \times 10^{11} \text{ V/m}$	CODATA 2022, $\sigma = -4.22$
$A_{ef} = 5.14220674763(78) \times 10^{11} \text{ V/m}$	CODATA 2018, $\sigma = +0.15$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = +4.47$

**atomic unit of force**

-16.0172(16)

$$A_{\text{force}} = \kappa_1^6 \left( \frac{l_p m_e^2}{t_p^2 m_p} \right) \left( 1 + 18 \sqrt{\frac{s^8}{4 (2\pi)^8}} (\kappa_1 + \kappa_3 + \kappa_4) \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $m_p$  = the Planck mass,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{force}} = 8.23872349803771 \dots \times 10^{-8} \text{ N}$	prediction
$A_{\text{force}} = 8.2387235038(13) \times 10^{-8} \text{ N}$	CODATA 2022, $\sigma = -4.43$
$A_{\text{force}} = 8.2387234983(12) \times 10^{-8} \text{ N}$	CODATA 2018, $\sigma = -0.22$
	$\Delta_{\text{precision}} = -0.04$
	$\Delta_{\text{scaled}} = +4.58$

**atomic unit of charge density**

-25.9362(45)

$$A_{\text{cd}} = \kappa_1^7 \left( \frac{q_p m_e^3}{l_p^3 m_p^3} \right) \left( 1 + 18 \sqrt{\frac{s^9}{8 (2\pi)^8}} (\kappa_1 + \kappa_3 + \kappa_4) \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $q_p$  = the Planck charge,  $m_e$  = the electron mass,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{cd}} = 1.08120238561797 \dots \times 10^{12} \text{ C/m}^3$	prediction
$A_{\text{cd}} = 1.08120238677(51) \times 10^{12} \text{ C/m}^3$	CODATA 2022, $\sigma = -2.26$
$A_{\text{cd}} = 1.08120238457(49) \times 10^{12} \text{ C/m}^3$	CODATA 2018, $\sigma = +2.13$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = +4.49$

**neutron magnetic moment**

441.5(25)

$$\mu_n = -\frac{\varkappa_1^2 \varkappa_2^2}{C_{\text{CFP}}} \left( \frac{l_p^2 q_p m_p}{t_p m_n} \right) \left( 1 + 18 (2\pi) (\varkappa_1 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $C_{\text{CFP}}$  = the real fixed point of the hyperbolic cotangent,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_n$  = the neutron mass,  $\pi$  = Archimedes' constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_n = -9.66235818798183 \dots \times 10^{-27} \text{ J/T}$	prediction
$\mu_n = -9.6623653(23) \times 10^{-27} \text{ J/T}$	CODATA 2022, $\sigma = +3.09$
$\mu_n = -9.6623651(23) \times 10^{-27} \text{ J/T}$	CODATA 2018, $\sigma = +3.01$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.09$

**neutron gyromagnetic ratio in MHz/T**

436.1(24)

$$\dot{\gamma}_n = \frac{\varkappa_1^2 \varkappa_2^2}{\pi C_{\text{CFP}}} \frac{1}{\text{MHz}} \left( \frac{q_p}{s m_n} \right) \left( 1 + 18 (2\pi) (\varkappa_1 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent, MHz = the megahertz,  $q_p$  = the Planck charge,  $s$  = the second,  $m_n$  = the neutron mass,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\dot{\gamma}_n = 2.91646871897931 \dots \times 10^1 \text{ MHz/T}$	prediction
$\dot{\gamma}_n = 2.91646935(69) \times 10^1 \text{ MHz/T}$	CODATA 2022, $\sigma = -0.91$
$\dot{\gamma}_n = 2.91646931(69) \times 10^1 \text{ MHz/T}$	CODATA 2018, $\sigma = -0.86$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.06$

**neutron gyromagnetic ratio**

436.1(24)

$$\gamma_n = \frac{2 \varkappa_1^2 \varkappa_2^2}{C_{\text{CFP}}} \left( \frac{q_p}{m_n} \right) \left( 1 + 18 (2\pi) (\varkappa_1 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $q_p$  = the Planck charge,  $m_n$  = the neutron mass,  $\pi$  = Archimedes' constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\gamma_n = 1.83247134039396 \dots \times 10^8 \text{ 1/s} \cdot \text{T}$	prediction
$\gamma_n = 1.83247174(43) \times 10^8 \text{ 1/s} \cdot \text{T}$	CODATA 2022, $\sigma = -0.93$
$\gamma_n = 1.83247171(43) \times 10^8 \text{ 1/s} \cdot \text{T}$	CODATA 2018, $\sigma = -0.86$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.07$

**neutron magnetic moment to Bohr magneton ratio**

435.7(24)

$$\frac{\mu_n}{\mu_B} = -\frac{2 \varkappa_1 \varkappa_2^2}{C_{\text{CFP}}} \left( \frac{m_e}{m_n} \right) \left( 1 + 18 (2\pi) (\varkappa_1 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $m_e$  = the electron mass,  $m_n$  = the neutron mass,  $\pi$  = Archimedes' constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_n/\mu_B = -1.04187546165272 \dots \times 10^{-3}$	prediction
$\mu_n/\mu_B = -1.04187565(25) \times 10^{-3}$	CODATA 2022, $\sigma = +0.75$
$\mu_n/\mu_B = -1.04187563(25) \times 10^{-3}$	CODATA 2018, $\sigma = +0.67$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.08$

**neutron magnetic moment to nuclear magneton ratio**

435.7(23)

$$\frac{\mu_n}{\mu_N} = -\frac{2 \varkappa_1 \varkappa_2^2}{C_{\text{CFP}}} \left( \frac{m_+}{m_n} \right) \left( 1 + 18 (2\pi) (\varkappa_1 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $m_+$  = the proton mass,  $m_n$  = the neutron mass,  $\pi$  = Archimedes' constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\mu_n/\mu_N = -1.91304241427759 \dots$$

prediction

$$\mu_n/\mu_N = -1.91304276(45)$$

CODATA 2022,  $\sigma = +0.77$ 

$$\mu_n/\mu_N = -1.91304273(45)$$

CODATA 2018,  $\sigma = +0.70$ 

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.07$$

**neutron g factor**

435.7(23)

$$g_n = -\frac{4 \varkappa_1 \varkappa_2^2}{C_{\text{CFP}}} \left( \frac{m_+}{m_n} \right) \left( 1 + 18 (2\pi) (\varkappa_1 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $m_+$  = the proton mass,  $m_n$  = the neutron mass,  $\pi$  = Archimedes' constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$g_n = -3.82608482855518 \dots$$

prediction

$$g_n = -3.82608552(90)$$

CODATA 2022,  $\sigma = +0.77$ 

$$g_n = -3.82608545(90)$$

CODATA 2018,  $\sigma = +0.69$ 

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.08$$

## hertz-kilogram relationship

12.37064(68)

$$\text{Hz} : \text{kg} = 2\pi \left( \frac{t_p m_p}{s} \right) \left( 1 + \frac{2}{C_{\text{CFP}}} (\varkappa_2 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $m_p$  = the Planck mass,  $s$  = the second,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \text{Hz} : \text{kg} &= 7.372497323\mathbf{69515} \dots \times 10^{-51} \text{ kg} && \text{prediction} \\ \text{Hz} : \text{kg} &= 7.372497323 \times 10^{-51} \text{ kg} && \text{CODATA 2022, 10-digit match} \\ \text{Hz} : \text{kg} &= 7.372497323 \times 10^{-51} \text{ kg} && \text{CODATA 2018, 10-digit match} \\ &&& \Delta_{\text{precision}} = +0.00 \\ &&& \Delta_{\text{scaled}} = +0.00 \end{aligned}$$

## hertz-atomic mass unit relationship

12.3711(29)

$$\text{Hz} : A_{\text{mass}} = 2\pi \left( \frac{t_p m_p}{s A_{\text{mass}}} \right) \left( 1 + \frac{2}{C_{\text{CFP}}} (\varkappa_2 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $m_p$  = the Planck mass,  $s$  = the second,  $A_{\text{mass}}$  = the atomic mass constant,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \text{Hz} : A_{\text{mass}} &= 4.4398216\mathbf{6082254} \dots \times 10^{-24} \text{ u} && \text{prediction} \\ \text{Hz} : A_{\text{mass}} &= 4.4398216590(14) \times 10^{-24} \text{ u} && \text{CODATA 2022, } \sigma = +1.30 \\ \text{Hz} : A_{\text{mass}} &= 4.4398216652(13) \times 10^{-24} \text{ u} && \text{CODATA 2018, } \sigma = -3.37 \\ &&& \Delta_{\text{precision}} = -0.03 \\ &&& \Delta_{\text{scaled}} = -4.77 \end{aligned}$$

**natural unit of time**

12.3711(30)

$$\mathcal{N}_{\text{time}} = \left( \frac{t_p m_p}{m_e} \right) \left( 1 + \frac{2}{C_{\text{CFP}}} (\mathfrak{K}_2 - \mathfrak{K}_3 - \mathfrak{K}_4) \boxtimes \right)$$

Where  $t_p$  = the Planck time,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mathcal{N}_{\text{time}} = 1.28808866695581 \dots \times 10^{-21} \text{ s}$	prediction
$\mathcal{N}_{\text{time}} = 1.28808866644(40) \times 10^{-21} \text{ s}$	CODATA 2022, $\sigma = +1.29$
$\mathcal{N}_{\text{time}} = 1.28808866819(39) \times 10^{-21} \text{ s}$	CODATA 2018, $\sigma = -3.16$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = -4.49$

**atomic mass unit-hertz relationship**

-12.37064(68)

$$A_{\text{mass}} : \text{Hz} = \frac{1}{2\pi} \left( \frac{A_{\text{mass}}}{t_p m_p} \right) \left( 1 - \frac{2}{C_{\text{CFP}}} (\mathfrak{K}_2 - \mathfrak{K}_3 - \mathfrak{K}_4) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $A_{\text{mass}}$  = the atomic mass constant,  $t_p$  = the Planck time,  $m_p$  = the Planck mass,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{mass}} : \text{Hz} = 2.25234272093084 \dots \times 10^{23} \text{ Hz}$	prediction
$A_{\text{mass}} : \text{Hz} = 2.25234272185(70) \times 10^{23} \text{ Hz}$	CODATA 2022, $\sigma = -1.31$
$A_{\text{mass}} : \text{Hz} = 2.25234271871(68) \times 10^{23} \text{ Hz}$	CODATA 2018, $\sigma = +3.27$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = +4.61$

## kilogram-hertz relationship

-12.37064(68)

$$\text{kg} : \text{Hz} = \frac{1}{2\pi} \left( \frac{\text{kg}}{t_p m_p} \right) \left( 1 - \frac{2}{C_{\text{CFP}}} (\varkappa_2 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant, kg = the kilogram,  $t_p$  = the Planck time,  $m_p$  = the Planck mass,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\text{Kg} : \text{Hz} = 1.35639248967168 \dots \times 10^{50} \text{ Hz}$	prediction
$\text{kg} : \text{Hz} = 1.356392489 \times 10^{50} \text{ Hz}$	CODATA 2022, 10-digit match
$\text{kg} : \text{Hz} = 1.356392489 \times 10^{50} \text{ Hz}$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**Boltzmann constant**

47.18(73)

$$k_B = \left( \frac{l_p^2 m_p}{t_p^2 T_p} \right) \left( 1 + \frac{s}{\sqrt{2}} \left( \frac{2\pi}{4K} \right) (\varkappa_2 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$k_B = 1.38064900009246 \dots \times 10^{-23} \text{ J/K}$	prediction
$k_B = 1.380649 \times 10^{-23} \text{ J/K}$	CODATA 2022, 7-digit match
$k_B = 1.380649 \times 10^{-23} \text{ J/K}$	CODATA 2018, 7-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**Boltzmann constant in eV/K**

47.18940(98)

$$\ddot{k}_B = \frac{\text{joule}}{\text{eV}} \left( \frac{l_p^2 m_p}{t_p^2 T_p} \right) \left( 1 + \frac{s}{\sqrt{2}} \left( \frac{2\pi}{4K} \right) (\varkappa_2 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where eV = the electron-volt,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\ddot{k}_B = 8.61733326153815 \dots \times 10^{-5} \text{ eV/K}$	prediction
$\ddot{k}_B = 8.617333262 \times 10^{-5} \text{ eV/K}$	CODATA 2022, 9.50-digit match
$\ddot{k}_B = 8.617333262 \times 10^{-5} \text{ eV/K}$	CODATA 2018, 9.50-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**kelvin-joule relationship**

47.18(73)

$$K : J = \left( \frac{l_p^2 m_p K}{t_p^2 T_p} \right) \left( 1 + \frac{s}{\sqrt{2}} \left( \frac{2\pi}{4K} \right) (\mathfrak{K}_2 - \mathfrak{K}_3 - \mathfrak{K}_4) \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $K$  = the kelvin,  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

K : J = 1.38064900009246 ... × 10 <sup>-23</sup> J	prediction
K : J = 1.380649 × 10 <sup>-23</sup> J	CODATA 2022, 7-digit match
K : J = 1.380649 × 10 <sup>-23</sup> J	CODATA 2018, 7-digit match
	Δ <sub>precision</sub> = +0.00
	Δ <sub>scaled</sub> = +0.00

**kelvin-electron volt relationship**

47.18942(58)

$$K : eV = \frac{\text{joule}}{eV} \left( \frac{l_p^2 m_p K}{t_p^2 T_p} \right) \left( 1 + \frac{s}{\sqrt{2}} \left( \frac{2\pi}{4K} \right) (\mathfrak{K}_2 - \mathfrak{K}_3 - \mathfrak{K}_4) \boxtimes \right)$$

Where eV = the electron-volt,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $K$  = the kelvin,  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

K : eV = 8.61733326153815 ... × 10 <sup>-5</sup> eV	prediction
K : eV = 8.617333262 × 10 <sup>-5</sup> eV	CODATA 2022, 9.50-digit match
K : eV = 8.617333262 × 10 <sup>-5</sup> eV	CODATA 2018, 9.50-digit match
	Δ <sub>precision</sub> = +0.00
	Δ <sub>scaled</sub> = +0.00

## electron volt-kelvin relationship

-47.18942(58)

$$eV : K = eV \left( \frac{t_p^2 T_p}{l_p^2 m_p} \right) \left( 1 - \frac{s}{\sqrt{2}} \left( \frac{2\pi}{4K} \right) (\varkappa_2 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $eV$  = the electron-volt,  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$eV : K = 1.16045181221091 \dots \times 10^4 K$	prediction
$eV : K = 1.160451812 \times 10^4 K$	CODATA 2022, 10-digit match
$eV : K = 1.160451812 \times 10^4 K$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

## joule-kelvin relationship

-47.18675(69)

$$J : K = \text{joule} \left( \frac{t_p^2 T_p}{l_p^2 m_p} \right) \left( 1 - \frac{s}{\sqrt{2}} \left( \frac{2\pi}{4K} \right) (\varkappa_2 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$J : K = 7.24297051539357 \dots \times 10^{22} K$	prediction
$J : K = 7.242970516 \times 10^{22} K$	CODATA 2022, 9.92-digit match
$J : K = 7.242970516 \times 10^{22} K$	CODATA 2018, 9.92-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**Loschmidt constant (273.15 K, 100 kPa)**

-47.1889(19)

$$n_0 = \frac{p_0}{T_0} \left( \frac{t_p^2 T_p}{l_p^2 m_p} \right) \left( 1 - \frac{s}{\sqrt{2}} \left( \frac{2\pi}{4K} \right) (\varkappa_2 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $p_0 = 100.000000000000$  kPa,  $T_0 = 273.150000000000$  K,  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$n_0 = 2.65164580464711 \dots \times 10^{25} \text{ 1/m}^3$	prediction
$n_0 = 2.651645804 \times 10^{25} \text{ 1/m}^3$	CODATA 2022, 10-digit match
$n_0 = 2.651645804 \times 10^{25} \text{ 1/m}^3$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**Loschmidt constant (273.15 K, 101.325 kPa)**

-47.1885(19)

$$n_1 = \frac{p_1}{T_0} \left( \frac{t_p^2 T_p}{l_p^2 m_p} \right) \left( 1 - \frac{s}{\sqrt{2}} \left( \frac{2\pi}{4K} \right) (\varkappa_2 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $p_1 = 101.325000000000$  kPa,  $T_0 = 273.150000000000$  K,  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $K$  = Catalan's constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$n_1 = 2.686780111155868 \dots \times 10^{25} \text{ 1/m}^3$	prediction
$n_1 = 2.686780111 \times 10^{25} \text{ 1/m}^3$	CODATA 2022, 10-digit match
$n_1 = 2.686780111 \times 10^{25} \text{ 1/m}^3$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**Bohr magneton in K/T**

-41.6669(30)

$$\frac{\mu_B}{k_B} = \frac{\varkappa_1}{2} \left( \frac{t_p q_p T_p}{m_e} \right) \left( 1 - \sqrt{\frac{32}{G_{Gi}}} (\varkappa_2 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $T_p$  = the Planck temperature,  $m_e$  = the electron mass,  $G_{Gi}$  = Gieseking's constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_B/k_B = 6.71713814961807 \dots \times 10^{-1}$ K/T	prediction
$\mu_B/k_B = 6.7171381472(21) \times 10^{-1}$ K/T	CODATA 2022, $\sigma = +1.15$
$\mu_B/k_B = 6.7171381563(20) \times 10^{-1}$ K/T	CODATA 2018, $\sigma = -3.34$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.55$

**nuclear magneton in K/T**

-41.6669(30)

$$\frac{\mu_N}{k_B} = \frac{\varkappa_1}{2} \left( \frac{t_p q_p T_p}{m_+} \right) \left( 1 - \sqrt{\frac{32}{G_{Gi}}} (\varkappa_2 - \varkappa_3 - \varkappa_4) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $T_p$  = the Planck temperature,  $m_+$  = the proton mass,  $G_{Gi}$  = Gieseking's constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_N/k_B = 3.65826777199876 \dots \times 10^{-4}$ K/T	prediction
$\mu_N/k_B = 3.6582677706(11) \times 10^{-4}$ K/T	CODATA 2022, $\sigma = +1.27$
$\mu_N/k_B = 3.6582677756(11) \times 10^{-4}$ K/T	CODATA 2018, $\sigma = -3.27$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -4.55$

**Rydberg constant times hc in J**

-7.243621(11)

$$R_{\infty} \ddot{h}c = \frac{\varkappa_1^4}{2} \left( \frac{l_p^2 m_e}{t_p^2} \right) \left( 1 + \sqrt{35} (8 D_d) (\varkappa_2 + \varkappa_3 + \varkappa_4) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $D_d$  = the dimer constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$R_{\infty} \ddot{h}c = 2.17987236111219 \dots \times 10^{-18} \text{ J} \quad \text{prediction}$$

$$R_{\infty} \ddot{h}c = 2.1798723611030(24) \times 10^{-18} \text{ J} \quad \text{CODATA 2022, } \sigma = +3.83$$

$$R_{\infty} \ddot{h}c = 2.1798723611035(42) \times 10^{-18} \text{ J} \quad \text{CODATA 2018, } \sigma = +2.07$$

$$\Delta_{\text{precision}} = +0.24$$

$$\Delta_{\text{scaled}} = -0.12$$

**Hartree energy**

-7.243622(11)

$$E_h = \varkappa_1^4 \left( \frac{l_p^2 m_e}{t_p^2} \right) \left( 1 + \sqrt{35} (8 D_d) (\varkappa_2 + \varkappa_3 + \varkappa_4) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $D_d$  = the dimer constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$E_h = 4.35974472222439 \dots \times 10^{-18} \text{ J} \quad \text{prediction}$$

$$E_h = 4.3597447222060(48) \times 10^{-18} \text{ J} \quad \text{CODATA 2022, } \sigma = +3.83$$

$$E_h = 4.3597447222071(85) \times 10^{-18} \text{ J} \quad \text{CODATA 2018, } \sigma = +2.03$$

$$\Delta_{\text{precision}} = +0.25$$

$$\Delta_{\text{scaled}} = -0.13$$

Also listed as the *atomic unit of energy*, and the *hartree-joule relationship*.

## joule-hartree relationship

7.243627(11)

$$J : E_h = \frac{\text{joule}^2}{\kappa_1^4} \left( \frac{t_p^2}{l_p^2 m_e} \right) \left( 1 + \sqrt{35} (8 D_d) (\kappa_2 + \kappa_3 + \kappa_4) \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $t_p$  = the Planck time,  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $D_d$  = the dimer constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} J : E_h &= 2.29371227838603 \dots \times 10^{17} E_h && \text{prediction} \\ J : E_h &= 2.2937122783969(25) \times 10^{17} E_h && \text{CODATA 2022, } \sigma = -4.35 \\ J : E_h &= 2.2937122783963(45) \times 10^{17} E_h && \text{CODATA 2018, } \sigma = -2.28 \\ &&& \Delta_{\text{precision}} = +0.26 \\ &&& \Delta_{\text{scaled}} = +0.13 \end{aligned}$$

**Josephson constant**

-4.7726(05)

$$K_J = \frac{\varkappa_1}{\pi} \left( \frac{t_p q_p}{l_p^2 m_p} \right) \left( 1 - \frac{7}{8} \left( \frac{8}{4\pi} \right)^2 (\varkappa_1^2 + \varkappa_2^2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $\pi$  = Archimedes' constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$K_J = 4.835978484\mathbf{24072} \dots \times 10^{14} \text{ Hz/V} \quad \text{prediction}$$

$$K_J = 4.835978484 \times 10^{14} \text{ Hz/V} \quad \text{CODATA 2022, 10-digit match}$$

$$K_J = 4.835978484 \times 10^{14} \text{ Hz/V} \quad \text{CODATA 2018, 10-digit match}$$

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.00$$

Also listed as the *conventional value of Josephson constant*.

**elementary charge over h-bar**

-4.7771(14)

$$\frac{e}{\hbar} = \varkappa_1 \left( \frac{t_p q_p}{l_p^2 m_p} \right) \left( 1 - \frac{7}{8} \left( \frac{8}{4\pi} \right)^2 (\varkappa_1^2 + \varkappa_2^2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $\pi$  = Archimedes' constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$e/\hbar = 1.519267447\mathbf{90089} \dots \times 10^{15} \text{ A/J} \quad \text{prediction}$$

$$e/\hbar = 1.519267447 \times 10^{15} \text{ A/J} \quad \text{CODATA 2022, 10-digit match}$$

$$e/\hbar = 1.519267447 \times 10^{15} \text{ A/J} \quad \text{CODATA 2018, 10-digit match}$$

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.00$$

**magnetic flux quantum**

4.7717(12)

$$\Phi_0 = \frac{\pi}{\kappa_1} \left( \frac{l_p^2 m_p}{t_p q_p} \right) \left( 1 + \frac{7}{8} \left( \frac{8}{4\pi} \right)^2 (\kappa_1^2 + \kappa_2^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\Phi_0 = 2.06783384843115 \dots \times 10^{-15}$ Wb	prediction
$\Phi_0 = 2.067833848 \times 10^{-15}$ Wb	CODATA 2022, 10-digit match
$\Phi_0 = 2.067833848 \times 10^{-15}$ Wb	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**reduced Planck constant in eV s**

4.77270(38)

$$\frac{\hbar}{e} = \frac{1}{\kappa_1} \left( \frac{l_p^2 C m_p}{t_p q_p} \right) \left( 1 + \frac{7}{8} \left( \frac{8}{4\pi} \right)^2 (\kappa_1^2 + \kappa_2^2) \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $C$  = the coulomb,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $\pi$  = Archimedes' constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\hbar/e = 6.58211956941109 \dots \times 10^{-16}$ eV · s	prediction
$\hbar/e = 6.582119569 \times 10^{-16}$ eV · s	CODATA 2022, 10-digit match
$\hbar/e = 6.582119569 \times 10^{-16}$ eV · s	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

## Planck constant in eV/Hz

4.77117(60)

$$\frac{h}{e} = \frac{2\pi}{\varkappa_1} \left( \frac{l_p^2 C m_p}{t_p q_p} \right) \left( 1 + \frac{7}{8} \left( \frac{8}{4\pi} \right)^2 (\varkappa_1^2 + \varkappa_2^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $C$  = the coulomb,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\pi$  = Archimedes' constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$h/e = 4.13566769686229 \dots \times 10^{-15} \text{ eV/Hz} \quad \text{prediction}$$

$$h/e = 4.135667696 \times 10^{-15} \text{ eV/Hz} \quad \text{CODATA 2022, 10-digit match}$$

$$h/e = 4.135667696 \times 10^{-15} \text{ eV/Hz} \quad \text{CODATA 2018, 10-digit match}$$

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.00$$

**first radiation constant**

-2.0763(14)

$$c_1 = (2\pi)^2 \left( \frac{l_p^4 m_p}{t_p^3} \right) \left( 1 - \left( \frac{4\pi}{32} \right)^2 (\mathfrak{K}_1^2 + \mathfrak{K}_2^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $\mathfrak{K}_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$c_1 = 3.74177185285687 \dots \times 10^{-16} \text{ W} \cdot \text{m}^2$	prediction
$c_1 = 3.741771852 \times 10^{-16} \text{ W} \cdot \text{m}^2$	CODATA 2022, 10-digit match
$c_1 = 3.741771852 \times 10^{-16} \text{ W} \cdot \text{m}^2$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**first radiation constant for spectral radiance**

-2.0763(42)

$$c_{1L} = 4\pi \left( \frac{l_p^4 m_p}{t_p^3} \right) \left( 1 - \left( \frac{4\pi}{32} \right)^2 (\mathfrak{K}_1^2 + \mathfrak{K}_2^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $\mathfrak{K}_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$c_{1L} = 1.19104297260858 \dots \times 10^{-16} \text{ W} \cdot \text{m}^2/\text{sr}$	prediction
$c_{1L} = 1.191042972 \times 10^{-16} \text{ W} \cdot \text{m}^2/\text{sr}$	CODATA 2022, 10-digit match
$c_{1L} = 1.191042972 \times 10^{-16} \text{ W} \cdot \text{m}^2/\text{sr}$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**shielded helion gyromagnetic ratio**

-3.9241(88)

$$\gamma_{\text{he}}' = \frac{C_{\text{CFP}}^2}{\sqrt{7}} \left( \frac{q_p}{m_{\text{he}}} \right) \left( 1 - C_d (\kappa_1^2 + \kappa_2^2) \boxtimes \right)$$

Where  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $q_p$  = the Planck charge,  $m_{\text{he}}$  = the helion mass,  $C_d$  = the domino tiling constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \gamma_{\text{he}}' &= 2.03789460878416 \dots \times 10^8 \text{ 1/s} \cdot \text{T} && \text{prediction} \\ \gamma_{\text{he}}' &= 2.0378946078(18) \times 10^8 \text{ 1/s} \cdot \text{T} && \text{CODATA 2022, } \sigma = +0.55 \\ \gamma_{\text{he}}' &= 2.037894569(24) \times 10^8 \text{ 1/s} \cdot \text{T} && \text{CODATA 2018, } \sigma = +1.66 \\ &&& \Delta_{\text{precision}} = +1.13 \\ &&& \Delta_{\text{scaled}} = +1.58 \end{aligned}$$

**shielded helion gyromagnetic ratio in MHz/T**

-3.9241(88)

$$\dot{\gamma}_{\text{he}}' = \frac{C_{\text{CFP}}^2}{2\pi\sqrt{7} \text{ MHz}} \left( \frac{q_p}{s m_{\text{he}}} \right) \left( 1 - C_d (\kappa_1^2 + \kappa_2^2) \boxtimes \right)$$

Where  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $\pi$  = Archimedes' constant, MHz = the megahertz,  $q_p$  = the Planck charge,  $s$  = the second,  $m_{\text{he}}$  = the helion mass,  $C_d$  = the domino tiling constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \dot{\gamma}_{\text{he}}' &= 3.24341000488323 \dots \times 10^1 \text{ MHz/T} && \text{prediction} \\ \dot{\gamma}_{\text{he}}' &= 3.2434100033(28) \times 10^1 \text{ MHz/T} && \text{CODATA 2022, } \sigma = +0.57 \\ \dot{\gamma}_{\text{he}}' &= 3.243409942(38) \times 10^1 \text{ MHz/T} && \text{CODATA 2018, } \sigma = +1.65 \\ &&& \Delta_{\text{precision}} = +0.13 \\ &&& \Delta_{\text{scaled}} = +1.61 \end{aligned}$$

## helion magnetic moment

600.9249(87)

$$\mu_{\text{he}} = -\frac{C_{\text{CFP}}^2}{2\sqrt{7}} \left( \frac{l_p^2 q_p m_p}{t_p m_{\text{he}}} \right) \left( 1 + 5 \left( \frac{16}{D_d} \right) (\varkappa_1^2 + \varkappa_2^2) \boxtimes \right)$$

Where  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_{\text{he}}$  = the helion mass,  $D_d$  = the dimer constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \mu_{\text{he}} &= -1.07461755420270 \dots \times 10^{-26} \text{ J/T} && \text{prediction} \\ \mu_{\text{he}} &= -1.07461755198(93) \times 10^{-26} \text{ J/T} && \text{CODATA 2022, } \sigma = -2.39 \\ \mu_{\text{he}} &= -1.074617532(13) \times 10^{-26} \text{ J/T} && \text{CODATA 2018, } \sigma = -1.71 \\ &&& \Delta_{\text{precision}} = +1.15 \\ &&& \Delta_{\text{scaled}} = +1.38 \end{aligned}$$

## shielded helion magnetic moment

1.2231(87)

$$\mu_{\text{he}}' = -\frac{C_{\text{CFP}}^2}{2\sqrt{7}} \left( \frac{l_p^2 q_p m_p}{t_p m_{\text{he}}} \right) \left( 1 + \frac{1}{7} \left( \frac{8}{4\pi} \right) (\varkappa_1^2 + \varkappa_2^2) \boxtimes \right)$$

Where  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $m_{\text{he}}$  = the helion mass,  $\pi$  = Archimedes' constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \mu_{\text{he}}' &= -1.07455311098442 \dots \times 10^{-26} \text{ J/T} && \text{prediction} \\ \mu_{\text{he}}' &= -1.07455311035(93) \times 10^{-26} \text{ J/T} && \text{CODATA 2022, } \sigma = +0.68 \\ \mu_{\text{he}}' &= -1.074553090(13) \times 10^{-26} \text{ J/T} && \text{CODATA 2018, } \sigma = +1.61 \\ &&& \Delta_{\text{precision}} = +1.15 \\ &&& \Delta_{\text{scaled}} = +1.54 \end{aligned}$$

**shielded helion to proton magnetic moment ratio**

320.1798(87)

$$\frac{\mu_{\text{he}}'}{\mu_+} = -\frac{C_{\text{CFP}}^2}{2\sqrt{7}} \frac{S}{\varkappa_1^2 \varkappa_2^2} \left( \frac{m_+}{m_{\text{he}}} \right) \left( 1 - \sqrt{12} L^2 (\varkappa_1^2 - \varkappa_2^2) \boxtimes \right)$$

Where  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $S$  = Sierpiński's constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $m_+$  = the proton mass,  $m_{\text{he}}$  = the helion mass,  $L$  = the lemniscate constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_{\text{he}}'/\mu_+ = -7.617665777306786 \dots \times 10^{-1}$	prediction
$\mu_{\text{he}}'/\mu_+ = -7.6176657721(66) \times 10^{-1}$	CODATA 2022, $\sigma = -0.15$
$\mu_{\text{he}}'/\mu_+ = -7.617665618(89) \times 10^{-1}$	CODATA 2018, $\sigma = +1.74$
	$\Delta_{\text{precision}} = +1.13$
	$\Delta_{\text{scaled}} = +1.73$

**shielded helion to shielded proton magnetic moment ratio**

576.910(41)

$$\frac{\mu_{\text{he}}'}{\mu_+'} = -\frac{C_{\text{CFP}}^2}{2\sqrt{7}} \frac{S}{\varkappa_1^2 \varkappa_2^2} \left( \frac{m_+}{m_{\text{he}}} \right) \left( 1 + \frac{\pi}{\sqrt{12}} L^4 (\varkappa_1^2 + \varkappa_2^2) \boxtimes \right)$$

Where  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $S$  = Sierpiński's constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $m_+$  = the proton mass,  $m_{\text{he}}$  = the helion mass,  $\pi$  = Archimedes' constant,  $L$  = the lemniscate constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_{\text{he}}'/\mu_+' = -7.61786134617088 \dots \times 10^{-1}$	prediction
$\mu_{\text{he}}'/\mu_+' = -7.617861334(31) \times 10^{-1}$	CODATA 2022, $\sigma = -0.39$
$\mu_{\text{he}}'/\mu_+' = -7.617861313(33) \times 10^{-1}$	CODATA 2018, $\sigma = -1.00$
	$\Delta_{\text{precision}} = +0.03$
	$\Delta_{\text{scaled}} = +0.64$

**electron to shielded helion magnetic moment ratio**

654.1948(81)

$$\frac{\mu_e}{\mu_{\text{he}}'} = \frac{2\sqrt{7}}{C_{\text{CFP}}^2} \frac{\varkappa_1^2 \varkappa_2^2}{P_{up}} \left( \frac{m_{\text{he}}}{m_e} \right) \left( 1 + \frac{2e^\gamma}{\sqrt{12}} L^4 (\varkappa_1^2 + \varkappa_2^2) \boxtimes \right)$$

Where  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $m_{\text{he}}$  = the helion mass,  $m_e$  = the electron mass,  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $L$  = the lemniscate constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\mu_e/\mu_{\text{he}}' = 8.64058236541607 \dots \times 10^2$	prediction
$\mu_e/\mu_{\text{he}}' = 8.6405823986(70) \times 10^2$	CODATA 2022, $\sigma = -4.74$
$\mu_e/\mu_{\text{he}}' = 8.64058257(10) \times 10^2$	CODATA 2018, $\sigma = -2.05$
	$\Delta_{\text{precision}} = +1.15$
	$\Delta_{\text{scaled}} = -2.80$

**atomic mass unit-hartree relationship**

-1.3473(31)

$$A_{\text{mass}} : E_h = \sqrt{\frac{18}{32} \frac{P_{up}}{\mathfrak{K}_1^{14}}} \left( 1 + \frac{\zeta(3)}{12} (\mathfrak{K}_1^2 - \mathfrak{K}_2^2) \boxtimes \right)$$

Where  $P_{up}$  = the universal parabolic constant,  $\mathfrak{K}_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\zeta(x)$  = the Riemann zeta function,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{mass}} : E_h = 3.42317769077001 \dots \times 10^7 E_h$	prediction
$A_{\text{mass}} : E_h = 3.4231776922(11) \times 10^7 E_h$	CODATA 2022, $\sigma = -1.30$
$A_{\text{mass}} : E_h = 3.4231776874(10) \times 10^7 E_h$	CODATA 2018, $\sigma = +3.37$
	$\Delta_{\text{precision}} = -0.04$
	$\Delta_{\text{scaled}} = +4.80$

**hartree-atomic mass unit relationship**

1.3473(31)

$$E_h : A_{\text{mass}} = \sqrt{\frac{32}{18} \frac{\mathfrak{K}_1^{14}}{P_{up}}} \left( 1 - \frac{\zeta(3)}{12} (\mathfrak{K}_1^2 - \mathfrak{K}_2^2) \boxtimes \right)$$

Where  $\mathfrak{K}_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $\zeta(x)$  = the Riemann zeta function,  $\mathfrak{K}_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_h : A_{\text{mass}} = 2.92126231920798 \dots \times 10^{-8} \text{ u}$	prediction
$E_h : A_{\text{mass}} = 2.92126231797(91) \times 10^{-8} \text{ u}$	CODATA 2022, $\sigma = +1.36$
$E_h : A_{\text{mass}} = 2.92126232205(88) \times 10^{-8} \text{ u}$	CODATA 2018, $\sigma = -3.23$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.64$

## kilogram-hartree relationship

-1.346674(10)

$$\text{kg} : E_h = \sqrt{\frac{18}{32} \frac{P_{up}}{\varkappa_1^{14}} \left( \frac{\text{kg}}{A_{\text{mass}}} \right)} \left( 1 + \frac{\zeta(3)}{12} (\varkappa_1^2 - \varkappa_2^2) \boxtimes \right)$$

Where  $P_{up}$  = the universal parabolic constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\text{kg}$  = the kilogram,  $A_{\text{mass}}$  = the atomic mass constant,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\text{kg} : E_h = 2.06148578874053 \dots \times 10^{34} E_h$	prediction
$\text{kg} : E_h = 2.0614857887415(22) \times 10^{34} E_h$	CODATA 2022, $\sigma = -0.44$
$\text{kg} : E_h = 2.0614857887409(40) \times 10^{34} E_h$	CODATA 2018, $\sigma = -0.09$
	$\Delta_{\text{precision}} = +0.26$
	$\Delta_{\text{scaled}} = +0.15$

## hartree-kilogram relationship

1.346675(11)

$$E_h : \text{kg} = \sqrt{\frac{32}{18} \frac{\varkappa_1^{14}}{P_{up}} (A_{\text{mass}})} \left( 1 - \frac{\zeta(3)}{12} (\varkappa_1^2 - \varkappa_2^2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $P_{up}$  = the universal parabolic constant,  $A_{\text{mass}}$  = the atomic mass constant,  $\zeta(x)$  = the Riemann zeta function,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_h : \text{kg} = 4.85087020954402 \dots \times 10^{-35} \text{kg}$	prediction
$E_h : \text{kg} = 4.8508702095419(53) \times 10^{-35} \text{kg}$	CODATA 2022, $\sigma = +0.40$
$E_h : \text{kg} = 4.8508702095432(94) \times 10^{-35} \text{kg}$	CODATA 2018, $\sigma = +0.09$
	$\Delta_{\text{precision}} = +0.25$
	$\Delta_{\text{scaled}} = -0.14$

**kelvin-kilogram relationship**

54.41184(77)

$$K : \text{kg} = \left( \frac{K m_p}{T_p} \right) \left( 1 - L_2 \sqrt{6V_{\text{fe}}} (\varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

Where  $K$  = the kelvin,  $m_p$  = the Planck mass,  $T_p$  = the Planck temperature,  $L_2$  = the 2<sup>nd</sup> lemniscate constant,  $V_{\text{fe}}$  = the figure-eight knot hyperbolic volume,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

K : kg = 1.53617918688372 ... × 10 <sup>-40</sup> kg	prediction
K : kg = 1.536179187 × 10 <sup>-40</sup> kg	CODATA 2022, 9.93-digit match
K : kg = 1.536179187 × 10 <sup>-40</sup> kg	CODATA 2018, 9.93-digit match
	Δ <sub>precision</sub> = +0.00
	Δ <sub>scaled</sub> = +0.00

**kilogram-kelvin relationship**

-54.41184(77)

$$\text{kg} : K = \left( \frac{T_p \text{kg}}{m_p} \right) \left( 1 + L_2 \sqrt{6V_{\text{fe}}} (\varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

Where  $T_p$  = the Planck temperature,  $m_p$  = the Planck mass,  $L_2$  = the 2<sup>nd</sup> lemniscate constant,  $V_{\text{fe}}$  = the figure-eight knot hyperbolic volume,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

kg : K = 6.50965726204757 ... × 10 <sup>39</sup> K	prediction
kg : K = 6.509657260 × 10 <sup>39</sup> K	CODATA 2022, 9.50-digit match
kg : K = 6.509657260 × 10 <sup>39</sup> K	CODATA 2018, 9.50-digit match
	Δ <sub>precision</sub> = +0.00
	Δ <sub>scaled</sub> = +0.00

**kelvin-atomic mass unit relationship**

54.4119(31)

$$K : A_{\text{mass}} = \left( \frac{K m_p}{T_p A_{\text{mass}}} \right) \left( 1 - L_2 \sqrt{6V_{\text{fe}}} ( \varkappa_3^2 + \varkappa_4^2 ) \boxtimes \right)$$

Where  $A_{\text{mass}}$  = the atomic mass constant,  $m_p$  = the Planck mass,  $T_p$  = the Planck temperature,  $L_2$  = the 2<sup>nd</sup> lemniscate constant,  $V_{\text{fe}}$  = the figure-eight knot hyperbolic volume,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

K : $A_{\text{mass}} = 9.25108729020561 \dots \times 10^{-14}$ u	prediction
K : $A_{\text{mass}} = 9.2510872884(29) \times 10^{-14}$ u	CODATA 2022, $\sigma = +0.62$
K : $A_{\text{mass}} = 9.2510873014(28) \times 10^{-14}$ u	CODATA 2018, $\sigma = -4.00$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.64$

**atomic mass unit-kelvin relationship**

-54.4119(31)

$$A_{\text{mass}} : K = \left( \frac{T_p A_{\text{mass}}}{m_p} \right) \left( 1 + L_2 \sqrt{6V_{\text{fe}}} ( \varkappa_3^2 + \varkappa_4^2 ) \boxtimes \right)$$

Where  $T_p$  = the Planck temperature,  $A_{\text{mass}}$  = the atomic mass constant,  $m_p$  = the Planck mass,  $L_2$  = the 2<sup>nd</sup> lemniscate constant,  $V_{\text{fe}}$  = the figure-eight knot hyperbolic volume,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

A <sub>mass</sub> : K = 1.08095402043079 ... × 10 <sup>13</sup> K	prediction
A <sub>mass</sub> : K = 1.08095402067(34) × 10 <sup>13</sup> K	CODATA 2022, $\sigma = -0.70$
A <sub>mass</sub> : K = 1.08095401916(33) × 10 <sup>13</sup> K	CODATA 2018, $\sigma = +3.58$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = +4.58$

**kilogram-inverse meter relationship**

8.7598(22)

$$\text{kg} : \frac{1}{\text{m}} = \frac{1}{2\pi} \left( \frac{\text{kg}}{l_p m_p} \right) \left( 1 - \frac{P_{up} i}{8} (\kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

Kg : 1/m = 4.52443833598084 ... × 10 <sup>41</sup> cycles/m	prediction
kg : 1/m = 4.524438335 × 10 <sup>41</sup> cycles/m	CODATA 2022, 10-digit match
kg : 1/m = 4.524438335 × 10 <sup>41</sup> cycles/m	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**atomic mass unit-inverse meter relationship**

-8.7591(30)

$$A_{\text{mass}} : \frac{1}{\text{m}} = \frac{1}{2\pi} \left( \frac{A_{\text{mass}}}{l_p m_p} \right) \left( 1 - \frac{P_{up} i}{8} (\kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $A_{\text{mass}}$  = the atomic mass constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

A <sub>mass</sub> : 1/m = 7.51300661861781 ... × 10 <sup>14</sup> cycles/m	prediction
A <sub>mass</sub> : 1/m = 7.5130066209(23) × 10 <sup>14</sup> cycles/m	CODATA 2022, $\sigma = +0.99$
A <sub>mass</sub> : 1/m = 7.5130066104(23) × 10 <sup>14</sup> cycles/m	CODATA 2018, $\sigma = +3.57$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +4.57$

**inverse meter-kilogram relationship**

8.7598(22)

$$\frac{1}{\text{m}} : \text{kg} = 2\pi \left( \frac{l_p m_p}{\text{m}} \right) \left( 1 + \frac{P_{up} i}{8} (\kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $\text{m}$  = the meter,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} 1/\text{m} : \text{kg} &= 2.210219094\mathbf{04020} \dots \times 10^{-42} \text{ kg} && \text{prediction} \\ 1/\text{m} : \text{kg} &= 2.210219094 \times 10^{-42} \text{ kg} && \text{CODATA 2022, 10-digit match} \\ 1/\text{m} : \text{kg} &= 2.210219094 \times 10^{-42} \text{ kg} && \text{CODATA 2018, 10-digit match} \\ &&& \Delta_{\text{precision}} = +0.00 \\ &&& \Delta_{\text{scaled}} = +0.00 \end{aligned}$$

**inverse meter-atomic mass unit relationship**

8.7591(30)

$$\frac{1}{\text{m}} : A_{\text{mass}} = 2\pi \left( \frac{l_p m_p}{\text{m} A_{\text{mass}}} \right) \left( 1 + \frac{P_{up} i}{8} (\kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $\text{m}$  = the meter,  $A_{\text{mass}}$  = the atomic mass constant,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} 1/\text{m} : A_{\text{mass}} &= 1.331025048\mathbf{64185} \dots \times 10^{-15} \text{ u} && \text{prediction} \\ 1/\text{m} : A_{\text{mass}} &= 1.33102504824(41) \times 10^{-15} \text{ u} && \text{CODATA 2022, } \sigma = +0.98 \\ 1/\text{m} : A_{\text{mass}} &= 1.33102505010(40) \times 10^{-15} \text{ u} && \text{CODATA 2018, } \sigma = -3.65 \\ &&& \Delta_{\text{precision}} = -0.01 \\ &&& \Delta_{\text{scaled}} = -4.65 \end{aligned}$$

## Compton wavelength

8.7590(30)

$$\lambda_c = 2\pi \left( \frac{l_p m_p}{m_e} \right) \left( 1 + \frac{P_{up} i}{8} (\varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \lambda_c &= 2.42631023609293 \dots \times 10^{-12} \text{ m/cycle} && \text{prediction} \\ \lambda_c &= 2.42631023538(76) \times 10^{-12} \text{ m/cycle} && \text{CODATA 2022, } \sigma = +0.94 \\ \lambda_c &= 2.42631023867(73) \times 10^{-12} \text{ m/cycle} && \text{CODATA 2018, } \sigma = -3.53 \\ &&& \Delta_{\text{precision}} = -0.02 \\ &&& \Delta_{\text{scaled}} = -4.63 \end{aligned}$$

## muon Compton wavelength

8.72(22)

$$\lambda_\mu = 2\pi \left( \frac{l_p m_p}{m_\mu} \right) \left( 1 + \frac{P_{up} i}{8} (\varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_\mu$  = the muon mass,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} \lambda_\mu &= 1.17344411764093 \dots \times 10^{-14} \text{ m/cycle} && \text{prediction} \\ \lambda_\mu &= 1.173444110(26) \times 10^{-14} \text{ m/cycle} && \text{CODATA 2022, } \sigma = +0.29 \\ \lambda_\mu &= 1.173444110(26) \times 10^{-14} \text{ m/cycle} && \text{CODATA 2018, } \sigma = +0.29 \\ &&& \Delta_{\text{precision}} = +0.00 \\ &&& \Delta_{\text{scaled}} = +0.00 \end{aligned}$$

**proton Compton wavelength**

8.7591(30)

$$\lambda_+ = 2\pi \left( \frac{l_p m_p}{m_+} \right) \left( 1 + \frac{P_{up} i}{8} (\varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_+$  = the proton mass,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\lambda_+ = 1.32140985399774 \dots \times 10^{-15}$ m/cycle	prediction
$\lambda_+ = 1.32140985360(41) \times 10^{-15}$ m/cycle	CODATA 2022, $\sigma = +0.97$
$\lambda_+ = 1.32140985539(40) \times 10^{-15}$ m/cycle	CODATA 2018, $\sigma = -3.48$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = -4.48$

**neutron Compton wavelength**

8.7601(56)

$$\lambda_n = 2\pi \left( \frac{l_p m_p}{m_n} \right) \left( 1 + \frac{P_{up} i}{8} (\varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_n$  = the neutron mass,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\lambda_n = 1.31959090499128 \dots \times 10^{-15}$ m/cycle	prediction
$\lambda_n = 1.31959090382(67) \times 10^{-15}$ m/cycle	CODATA 2022, $\sigma = +1.75$
$\lambda_n = 1.31959090581(75) \times 10^{-15}$ m/cycle	CODATA 2018, $\sigma = -1.09$
	$\Delta_{\text{precision}} = +0.05$
	$\Delta_{\text{scaled}} = -2.65$

**tau Compton wavelength**

35.(672)

$$\lambda_\tau = 2\pi \left( \frac{l_p m_p}{m_\tau} \right) \left( 1 + \frac{P_{up} i}{8} (\varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_\tau$  = the tau mass,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\lambda_\tau = 6.97770282157982 \dots \times 10^{-16}$ m/cycle	prediction
$\lambda_\tau = 6.97771(47) \times 10^{-16}$ m/cycle	CODATA 2022, $\sigma = -0.02$
$\lambda_\tau = 6.97771(47) \times 10^{-16}$ m/cycle	CODATA 2018, $\sigma = -0.02$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**reduced Compton wavelength**

8.7591(31)

$$\lambda_- = \left( \frac{l_p m_p}{m_e} \right) \left( 1 + \frac{P_{up} i}{8} (\varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\lambda_- = 3.86159267548653 \dots \times 10^{-13}$ m/cycle	prediction
$\lambda_- = 3.8615926744(12) \times 10^{-13}$ m/cycle	CODATA 2022, $\sigma = +0.91$
$\lambda_- = 3.8615926796(12) \times 10^{-13}$ m/cycle	CODATA 2018, $\sigma = -4.55$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -4.33$

Also listed as the *natural unit of length*.

**reduced muon Compton wavelength**

8.72(23)

$$\lambda_{\mu_-} = \left( \frac{l_p m_p}{m_{\mu}} \right) \left( 1 + \frac{P_{up} i}{8} (\mathfrak{K}_3^2 - \mathfrak{K}_4^2) \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_{\mu}$  = the muon mass,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\lambda_{\mu_-} = 1.86759431764662 \dots \times 10^{-15} \text{ m/cycle} \quad \text{prediction}$$

$$\lambda_{\mu_-} = 1.867594306(42) \times 10^{-15} \text{ m/cycle} \quad \text{CODATA 2022, } \sigma = +0.28$$

$$\lambda_{\mu_-} = 1.867594306(42) \times 10^{-15} \text{ m/cycle} \quad \text{CODATA 2018, } \sigma = +0.28$$

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.00$$

**reduced proton Compton wavelength**

8.7591(31)

$$\lambda_{+_-} = \left( \frac{l_p m_p}{m_+} \right) \left( 1 + \frac{P_{up} i}{8} (\mathfrak{K}_3^2 - \mathfrak{K}_4^2) \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_+$  = the proton mass,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\lambda_{+_-} = 2.10308910114080 \dots \times 10^{-16} \text{ m/cycle} \quad \text{prediction}$$

$$\lambda_{+_-} = 2.10308910051(66) \times 10^{-16} \text{ m/cycle} \quad \text{CODATA 2022, } \sigma = +0.96$$

$$\lambda_{+_-} = 2.10308910336(64) \times 10^{-16} \text{ m/cycle} \quad \text{CODATA 2018, } \sigma = -3.47$$

$$\Delta_{\text{precision}} = -0.01$$

$$\Delta_{\text{scaled}} = -4.45$$

**reduced neutron Compton wavelength**

8.7603(58)

$$\lambda_{n_-} = \left( \frac{l_p m_p}{m_n} \right) \left( 1 + \frac{P_{up} i}{8} (\varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_n$  = the neutron mass,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\lambda_{n_-} = 2.10019415388470 \dots \times 10^{-16}$ m/cycle	prediction
$\lambda_{n_-} = 2.1001941520(11) \times 10^{-16}$ m/cycle	CODATA 2022, $\sigma = +1.71$
$\lambda_{n_-} = 2.1001941552(12) \times 10^{-16}$ m/cycle	CODATA 2018, $\sigma = -1.10$
	$\Delta_{\text{precision}} = +0.04$
	$\Delta_{\text{scaled}} = -2.67$

**reduced tau Compton wavelength**

43.(675)

$$\lambda_{\tau_-} = \left( \frac{l_p m_p}{m_\tau} \right) \left( 1 + \frac{P_{up} i}{8} (\varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $m_\tau$  = the tau mass,  $P_{up}$  = the universal parabolic constant,  $i$  = the imaginary unit,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\lambda_{\tau_-} = 1.11053589548069 \dots \times 10^{-16}$ m/cycle	prediction
$\lambda_{\tau_-} = 1.110538(75) \times 10^{-16}$ m/cycle	CODATA 2022, $\sigma = -0.03$
$\lambda_{\tau_-} = 1.110538(75) \times 10^{-16}$ m/cycle	CODATA 2018, $\sigma = -0.03$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**Wien frequency displacement law constant**

42.04047(85)

$$b' = \frac{1}{2\pi} \left( \frac{\nu_{\text{peak}}}{t_p T_p} \right) \left( 1 + 18 \left( \frac{2\pi}{4s} \right)^2 (\kappa_1^2 - \kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

Where  $\nu_{\text{peak}}$  = the peak emission of spectral flux per unit frequency,  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $s$  = the arc length of the unit lemniscate,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$b' = 5.87892575796309 \dots \times 10^{10}$ Hz/K	prediction
$b' = 5.878925757 \times 10^{10}$ Hz/K	CODATA 2022, 10-digit match
$b' = 5.878925757 \times 10^{10}$ Hz/K	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

$\nu_{\text{peak}} = 2.82143937212207 \dots$  is the solution to the equation  $\frac{x e^x}{e^{x-1}} = 3$ .

**Boltzmann constant in Hz/K**

42.0407(24)

$$\ddot{k}_B = \frac{1}{2\pi} \left( \frac{1}{t_p T_p} \right) \left( 1 + 18 \left( \frac{2\pi}{4s} \right)^2 (\kappa_1^2 - \kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $s$  = the arc length of the unit lemniscate,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\ddot{k}_B = 2.08366191244486 \dots \times 10^{10}$ Hz/K	prediction
$\ddot{k}_B = 2.083661912 \times 10^{10}$ Hz/K	CODATA 2022, 10-digit match
$\ddot{k}_B = 2.083661912 \times 10^{10}$ Hz/K	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**kelvin-hertz relationship**

42.0414(24)

$$K : \text{Hz} = \frac{1}{2\pi} \left( \frac{K}{t_p T_p} \right) \left( 1 + 18 \left( \frac{2\pi}{4s} \right)^2 (\varkappa_1^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $K$  = the kelvin,  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $s$  = the arc length of the unit lemniscate,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

K : Hz = 2.08366191244486 ... × 10 <sup>10</sup> Hz	prediction
K : Hz = 2.083661912 × 10 <sup>10</sup> Hz	CODATA 2022, 10-digit match
K : Hz = 2.083661912 × 10 <sup>10</sup> Hz	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**hertz-kelvin relationship**

-42.0414(24)

$$\text{Hz} : K = 2\pi \left( \frac{t_p T_p}{s} \right) \left( 1 - 18 \left( \frac{2\pi}{4s} \right)^2 (\varkappa_1^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $s$  = the second,  $s$  = the arc length of the unit lemniscate,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

Hz : K = 4.79924307302320 ... × 10 <sup>-11</sup> K	prediction
Hz : K = 4.799243073 × 10 <sup>-11</sup> K	CODATA 2022, 10-digit match
Hz : K = 4.799243073 × 10 <sup>-11</sup> K	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**Wien wavelength displacement law constant**

-45.6561(28)

$$b = 2\pi \left( \frac{l_p T_p}{\lambda_{\text{peak}}} \right) \left( 1 + P^2 ( \varkappa_1^2 + \varkappa_3^2 + \varkappa_4^2 ) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $\lambda_{\text{peak}}$  = the peak wavelength of spectral radiance per unit wavelength,  $l_p$  = the Planck length,  $T_p$  = the Planck temperature,  $P$  = the plastic constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$b = 2.89777195379426 \dots \times 10^{-3} \text{ m} \cdot \text{K}/\text{cycle}$	prediction
$b = 2.897771955 \times 10^{-3} \text{ m} \cdot \text{K}/\text{cycle}$	CODATA 2022, 9-digit match
$b = 2.897771955 \times 10^{-3} \text{ m} \cdot \text{K}/\text{cycle}$	CODATA 2018, 9-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

$\lambda_{\text{peak}} = 4.96511423174427 \dots$  is the solution to the equation  $\frac{x e^x}{e^x - 1} = 5$ .

**second radiation constant**

-45.6561(28)

$$c_2 = 2\pi ( l_p T_p ) \left( 1 + P^2 ( \varkappa_1^2 + \varkappa_3^2 + \varkappa_4^2 ) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $T_p$  = the Planck temperature,  $P$  = the plastic constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$c_2 = 1.43877687681333 \dots \times 10^{-2} \text{ m} \cdot \text{K}$	prediction
$c_2 = 1.438776877 \times 10^{-2} \text{ m} \cdot \text{K}$	CODATA 2022, 10-digit match
$c_2 = 1.438776877 \times 10^{-2} \text{ m} \cdot \text{K}$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**inverse meter-kelvin relationship**

-45.6543(35)

$$\frac{1}{\text{m}} : \text{K} = 2\pi \left( \frac{l_p T_p}{\text{m}} \right) \left( 1 + P^2 ( \varkappa_1^2 + \varkappa_3^2 + \varkappa_4^2 ) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $T_p$  = the Planck temperature, m = the meter,  $P$  = the plastic constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

1/m : K = 1.43877687681333 ... × 10 <sup>-2</sup> K	prediction
1/m : K = 1.438776877 × 10 <sup>-2</sup> K	CODATA 2022, 10-digit match
1/m : K = 1.438776877 × 10 <sup>-2</sup> K	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**kelvin-inverse meter relationship**

45.6543(35)

$$\text{K} : \frac{1}{\text{m}} = \frac{1}{2\pi} \left( \frac{\text{K}}{l_p T_p} \right) \left( 1 - P^2 ( \varkappa_1^2 + \varkappa_3^2 + \varkappa_4^2 ) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant, K = the kelvin,  $l_p$  = the Planck length,  $T_p$  = the Planck temperature,  $P$  = the plastic constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

K : 1/m = 6.95034800805251 ... × 10 <sup>1</sup> cycles/m	prediction
K : 1/m = 6.950348004 × 10 <sup>1</sup> cycles/m	CODATA 2022, 9.23-digit match
K : 1/m = 6.950348004 × 10 <sup>1</sup> cycles/m	CODATA 2018, 9.23-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**Boltzmann constant in inverse meter per kelvin**

45.65224(72)

$$\dot{k}_B = \frac{1}{2\pi} \left( \frac{1}{l_p T_p} \right) \left( 1 - P^2 ( \varkappa_1^2 + \varkappa_3^2 + \varkappa_4^2 ) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $l_p$  = the Planck length,  $T_p$  = the Planck temperature,  $K_{-6}$  = the Khinchin mean of order  $-6$ ,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\dot{k}_B = 6.95034800805251 \dots \times 10^1 \text{ cycles/m} \cdot \text{K} \quad \text{prediction}$$

$$\dot{k}_B = 6.950348004 \times 10^1 \text{ cycles/m} \cdot \text{K} \quad \text{CODATA 2022, 9.23-digit match}$$

$$\dot{k}_B = 6.950348004 \times 10^1 \text{ cycles/m} \cdot \text{K} \quad \text{CODATA 2018, 9.23-digit match}$$

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.00$$

**hartree-kelvin relationship**

-54.431024(11)

$$E_h : K = \varkappa_1^4 \left( \frac{T_p m_e}{m_p} \right) \left( 1 + \frac{s}{\sqrt{2\pi}} \left( \varkappa_1^2 + \varkappa_3^2 + \varkappa_4^2 \right) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $T_p$  = the Planck temperature,  $m_e$  = the electron mass,  $m_p$  = the Planck mass,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$E_h : K = 3.15775024803817 \dots \times 10^5 \text{ K}$	prediction
$E_h : K = 3.1577502480398(34) \times 10^5 \text{ K}$	CODATA 2022, $\sigma = -0.48$
$E_h : K = 3.1577502480407(61) \times 10^5 \text{ K}$	CODATA 2018, $\sigma = -0.41$
	$\Delta_{\text{precision}} = +0.25$
	$\Delta_{\text{scaled}} = -0.15$

**kelvin-hartree relationship**

54.431320(11)

$$K : E_h = \frac{1}{\varkappa_1^4} \left( \frac{K m_p}{T_p m_e} \right) \left( 1 + \frac{s}{\sqrt{2\pi}} \left( \varkappa_1^2 + \varkappa_3^2 + \varkappa_4^2 \right) \boxtimes \right)^{-1}$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $K$  = the kelvin,  $m_p$  = the Planck mass,  $T_p$  = the Planck temperature,  $m_e$  = the electron mass,  $s$  = the arc length of the unit lemniscate,  $\pi$  = Archimedes' constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$K : E_h = 3.16681156345811 \dots \times 10^{-6} E_h$	prediction
$K : E_h = 3.1668115634564(35) \times 10^{-6} E_h$	CODATA 2022, $\sigma = +0.50$
$K : E_h = 3.1668115634556(61) \times 10^{-6} E_h$	CODATA 2018, $\sigma = +0.41$
	$\Delta_{\text{precision}} = +0.24$
	$\Delta_{\text{scaled}} = +0.13$

**electron molar mass**

-283.54(31)

$$M_e = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{C \text{ kg } m_e}{q_p m_p} \right) \left( 1 + 18 C_Q (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $\text{kg}$  = the kilogram,  $m_e$  = the electron mass,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $C_Q$  = the QRS constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} M_e &= 5.48579909\mathbf{796588} \dots \times 10^{-7} \text{ kg/mol} && \text{prediction} \\ M_e &= 5.4857990962(17) \times 10^{-7} \text{ kg/mol} && \text{CODATA 2022, } \sigma = +1.04 \\ M_e &= 5.4857990888(17) \times 10^{-7} \text{ kg/mol} && \text{CODATA 2018, } \sigma = +5.39 \\ &&& \Delta_{\text{precision}} = +0.00 \\ &&& \Delta_{\text{scaled}} = +4.35 \end{aligned}$$

**muon molar mass**

-283.50(22)

$$M_\mu = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{C \text{ kg } m_\mu}{q_p m_p} \right) \left( 1 + 18 C_Q (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $\text{kg}$  = the kilogram,  $m_\mu$  = the muon mass,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $C_Q$  = the QRS constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$\begin{aligned} M_\mu &= 1.13428925\mathbf{199289} \dots \times 10^{-4} \text{ kg/mol} && \text{prediction} \\ M_\mu &= 1.134289258(25) \times 10^{-4} \text{ kg/mol} && \text{CODATA 2022, } \sigma = -0.24 \\ M_\mu &= 1.134289259(25) \times 10^{-4} \text{ kg/mol} && \text{CODATA 2018, } \sigma = -0.28 \\ &&& \Delta_{\text{precision}} = +0.00 \\ &&& \Delta_{\text{scaled}} = -0.04 \end{aligned}$$

**molar mass constant**

-283.5449(30)

$$M_{A_{\text{mass}}} = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{C \text{ kg } A_{\text{mass}}}{q_p m_p} \right) \left( 1 + 18 C_Q (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $\text{kg}$  = the kilogram,  $A_{\text{mass}}$  = the atomic mass constant,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $C_Q$  = the QRS constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$M_{A_{\text{mass}}} = 1.00000000136176 \dots \times 10^{-3} \text{ kg/mol}$	prediction
$M_{A_{\text{mass}}} = 1.00000000105(31) \times 10^{-3} \text{ kg/mol}$	CODATA 2022, $\sigma = +1.01$
$M_{A_{\text{mass}}} = 0.99999999965(30) \times 10^{-3} \text{ kg/mol}$	CODATA 2018, $\sigma = +5.71$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = +4.67$

**proton molar mass**

-283.5447(30)

$$M_+ = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{C \text{ kg } m_+}{q_p m_p} \right) \left( 1 + 18 C_Q (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $\text{kg}$  = the kilogram,  $m_+$  = the proton mass,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $C_Q$  = the QRS constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$M_+ = 1.00727646795396 \dots \times 10^{-3} \text{ kg/mol}$	prediction
$M_+ = 1.00727646764(31) \times 10^{-3} \text{ kg/mol}$	CODATA 2022, $\sigma = +1.01$
$M_+ = 1.00727646627(31) \times 10^{-3} \text{ kg/mol}$	CODATA 2018, $\sigma = +5.43$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +4.42$

**neutron molar mass**

-283.5457(57)

$$M_n = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{C \text{ kg } m_n}{q_p m_p} \right) \left( 1 + 18 C_Q (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $\text{kg}$  = the kilogram,  $m_n$  = the neutron mass,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $C_Q$  = the QRS constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$M_n = 1.00866491684647 \dots \times 10^{-3} \text{ kg/mol}$	prediction
$M_n = 1.00866491712(51) \times 10^{-3} \text{ kg/mol}$	CODATA 2022, $\sigma = -0.54$
$M_n = 1.00866491560(57) \times 10^{-3} \text{ kg/mol}$	CODATA 2018, $\sigma = +2.19$
	$\Delta_{\text{precision}} = +0.05$
	$\Delta_{\text{scaled}} = +2.67$

**tau molar mass**

-287.(680)

$$M_\tau = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{C \text{ kg } m_\tau}{q_p m_p} \right) \left( 1 + 18 C_Q (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $\text{kg}$  = the kilogram,  $m_\tau$  = the tau mass,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $C_Q$  = the QRS constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$M_\tau = 1.90754046781407 \dots \times 10^{-3} \text{ kg/mol}$	prediction
$M_\tau = 1.90754(13) \times 10^{-3} \text{ kg/mol}$	CODATA 2022, $\sigma = +0.00$
$M_\tau = 1.90754(13) \times 10^{-3} \text{ kg/mol}$	CODATA 2018, $\sigma = +0.00$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**deuteron molar mass**

-283.5443(31)

$$M_{\text{de}} = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{C \text{ kg } m_{\text{de}}}{q_p m_p} \right) \left( 1 + 18 C_Q (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $\text{kg}$  = the kilogram,  $m_{\text{de}}$  = the deuteron mass,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $C_Q$  = the QRS constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$M_{\text{de}} = 2.01355321529080 \dots \times 10^{-3} \text{ kg/mol}$	prediction
$M_{\text{de}} = 2.01355321466(63) \times 10^{-3} \text{ kg/mol}$	CODATA 2022, $\sigma = +1.00$
$M_{\text{de}} = 2.01355321205(61) \times 10^{-3} \text{ kg/mol}$	CODATA 2018, $\sigma = +5.31$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = +4.28$

**helion molar mass**

-283.5447(30)

$$M_{\text{he}} = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{C \text{ kg } m_{\text{he}}}{q_p m_p} \right) \left( 1 + 18 C_Q (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $\text{kg}$  = the kilogram,  $m_{\text{he}}$  = the helion mass,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $C_Q$  = the QRS constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$M_{\text{he}} = 3.01493225103642 \dots \times 10^{-3} \text{ kg/mol}$	prediction
$M_{\text{he}} = 3.01493225010(94) \times 10^{-3} \text{ kg/mol}$	CODATA 2022, $\sigma = +1.00$
$M_{\text{he}} = 3.01493224613(91) \times 10^{-3} \text{ kg/mol}$	CODATA 2018, $\sigma = +5.39$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = +4.36$

**triton molar mass**

-283.5439(30)

$$M_{\text{tri}} = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{C \text{ kg } m_{\text{tri}}}{q_p m_p} \right) \left( 1 + 18 C_Q (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $\text{kg}$  = the kilogram,  $m_{\text{tri}}$  = the triton mass,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $C_Q$  = the QRS constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$M_{\text{tri}} = 3.01550071982806 \dots \times 10^{-3} \text{ kg/mol}$	prediction
$M_{\text{tri}} = 3.01550071913(94) \times 10^{-3} \text{ kg/mol}$	CODATA 2022, $\sigma = +0.74$
$M_{\text{tri}} = 3.01550071517(92) \times 10^{-3} \text{ kg/mol}$	CODATA 2018, $\sigma = +5.06$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = +4.30$

**alpha particle molar mass**

-283.5454(30)

$$M_\alpha = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{C \text{ kg } m_\alpha}{q_p m_p} \right) \left( 1 + 18 C_Q (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $\text{kg}$  = the kilogram,  $m_\alpha$  = the alpha particle mass,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $C_Q$  = the QRS constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$M_\alpha = 4.00150618457836 \dots \times 10^{-3} \text{ kg/mol}$	prediction
$M_\alpha = 4.0015061833(12) \times 10^{-3} \text{ kg/mol}$	CODATA 2022, $\sigma = +1.07$
$M_\alpha = 4.0015061777(12) \times 10^{-3} \text{ kg/mol}$	CODATA 2018, $\sigma = +5.73$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +4.67$

**molar mass of carbon-12**

-283.544(33)

$$M_{12_C} = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{12 C \text{ kg } A_{\text{mass}}}{q_p m_p} \right) \left( 1 + 18 C_Q (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $\text{kg}$  = the kilogram,  $A_{\text{mass}}$  = the atomic mass constant,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $C_Q$  = the QRS constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$M_{12_C} = 1.20000000163412 \dots \times 10^{-2} \text{ kg/mol} \quad \text{prediction}$$

$$M_{12_C} = 1.20000000126(36) \times 10^{-2} \text{ kg/mol} \quad \text{CODATA 2022, } \sigma = +1.04$$

$$M_{12_C} = 1.19999999958(36) \times 10^{-2} \text{ kg/mol} \quad \text{CODATA 2018, } \sigma = +5.71$$

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +4.67$$

**Avogadro constant**

-283.5364(83)

$$N_A = \frac{6}{e^\gamma} \kappa_1^2 \left( \frac{C \text{ kg}}{q_p m_p} \right) \left( 1 + 18 C_Q (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $e$  = Euler's number,  $\gamma$  = the Euler-Mascheroni constant,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $C$  = the coulomb,  $\text{kg}$  = the kilogram,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $C_Q$  = the QRS constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$N_A = 6.02214076443131 \dots \times 10^{23} \text{ 1/mol} \quad \text{prediction}$$

$$N_A = 6.02214076 \times 10^{23} \text{ 1/mol} \quad \text{CODATA 2022, 9.13-digit match}$$

$$N_A = 6.02214076 \times 10^{23} \text{ 1/mol} \quad \text{CODATA 2018, 9.13-digit match}$$

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.00$$

**Sackur-Tetrode constant (1K, 100 kPa)**

-28.0784(13)

$$ST_0 = \frac{5}{2} + \log \left( \left( \frac{1}{2\pi} \frac{K A_{\text{mass}}}{l_p^2 T_p m_p} \right)^{3/2} \frac{K E_p}{T_p p_0} \right) \left( 1 + j_{0,1} \left( \frac{2\pi}{14} \right) (\kappa_1^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $\log(x)$  = the natural logarithm,  $\pi$  = Archimedes' constant,  $K$  = kelvin,  $A_{\text{mass}}$  = the atomic mass constant,  $l_p$  = the Planck length,  $T_p$  = the Planck temperature,  $m_p$  = the Planck mass,  $E_p$  = the Planck energy,  $p_0$  = 100.000000000000 ... kPa,  $j_{0,1}$  = the 1<sup>st</sup> root of the Bessel function,  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$ST_0 = -1.15170753432430 \dots$$

prediction

$$ST_0 = -1.15170753496(47)$$

CODATA 2022,  $\sigma = +1.35$ 

$$ST_0 = -1.15170753706(45)$$

CODATA 2018,  $\sigma = -6.08$ 

$$\Delta_{\text{precision}} = -0.02$$

$$\Delta_{\text{scaled}} = -4.66$$

The Planck energy is equal to the product of the Planck length squared and the Planck mass, divided by the Planck time squared,

$$E_p = \frac{l_p^2 m_p}{t_p^2}$$

**Sackur-Tetrode constant (1K, 101.325 kPa)**

-27.9776(13)

$$ST_1 = \frac{5}{2} + \log \left( \left( \frac{1}{2\pi} \frac{K A_{\text{mass}}}{l_p^2 T_p m_p} \right)^{3/2} \frac{K E_p}{T_p p_1} \right) \left( 1 + \frac{1}{G_{Ga}} \left( \frac{4\pi}{14} \right) (\varkappa_1^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

Where  $\log(x)$  = the natural logarithm,  $\pi$  = Archimedes' constant,  $A_{\text{mass}}$  = the atomic mass constant,  $K$  = kelvin,  $l_p$  = the Planck length,  $m_p$  = the Planck mass,  $T_p$  = the Planck temperature,  $E_p$  = the Planck energy,  $p_1$  = 101.325000000000 kPa,  $G_{Ga}$  = Gauss's constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$ST_1 = -1.16487052045596 \dots$	prediction
$ST_1 = -1.16487052149(47)$	CODATA 2022, $\sigma = +2.20$
$ST_1 = -1.16487052358(45)$	CODATA 2018, $\sigma = -6.94$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = -4.64$

The Planck energy is equal to the product of the Planck length squared and the Planck mass, divided by the Planck time squared,

$$E_p = \frac{l_p^2 m_p}{t_p^2}$$

# Stefan-Boltzmann constant

180.53565(88)

$$\sigma = \frac{3}{5} \left( \frac{4\pi}{\Gamma(5)} \right)^2 \left( \frac{m_p}{t_p^3 T_p^4} \right) \left( 1 + (18 + 35) \left( \frac{2K}{14} \right) (\mathfrak{K}_1^2 - \mathfrak{K}_3^2 - \mathfrak{K}_4^2) \boxtimes \right)$$

Where  $\pi$  = Archimedes' constant,  $\Gamma(x)$  = the gamma function,  $m_p$  = the Planck mass,  $t_p$  = the Planck time,  $T_p$  = the Planck temperature,  $K$  = Catalan's constant,  $\mathfrak{K}_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\mathfrak{K}_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\mathfrak{K}_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$\sigma = 5.67037441987317 \dots \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$	prediction
$\sigma = 5.670374419 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$	CODATA 2022, 10-digit match
$\sigma = 5.670374419 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$	CODATA 2018, 10-digit match
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = +0.00$

**conventional value of volt-90**

1.066651(50)

$$V_{90} = \text{volt} \left( 1 + \frac{L_1}{32} ( \varkappa_1^2 - \varkappa_3^2 - \varkappa_4^2 ) \boxtimes \right)$$

Where  $L_1$  = the 1<sup>st</sup> lemniscate constant,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$V_{90} = 1.00000010665218 \dots V$$

prediction

$$V_{90} = 1.00000010666 V$$

CODATA 2022, 11.11-digit match

$$V_{90} = 1.00000010666 V$$

CODATA 2018, 11.11-digit match

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.00$$

**conventional value of watt-90**

1.955351(50)

$$W_{90} = \text{watt} \left( 1 - \frac{j_{0,1}}{32} ( \varkappa_1^2 + \varkappa_3^2 + \varkappa_4^2 ) \boxtimes \right)$$

Where  $j_{0,1}$  = the 1<sup>st</sup> root of the Bessel function,  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$$W_{90} = 1.00000019552285 \dots W$$

prediction

$$W_{90} = 1.00000019553 W$$

CODATA 2022, 11.15-digit match

$$W_{90} = 1.00000019553 W$$

CODATA 2018, 11.15-digit match

$$\Delta_{\text{precision}} = +0.00$$

$$\Delta_{\text{scaled}} = +0.00$$

**atomic unit of electric field gradient**

-25.1510(31)

$$A_{\text{ef}\nabla} = \varkappa_1^7 \left( \frac{m_e^3}{t_p^2 q_p m_p^2} \right) \left( 1 + \sqrt{35} \left( \frac{V_{\text{fe}}}{6} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $V_{\text{fe}}$  = the figure eight knot hyperbolic volume,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{ef}\nabla} = 9.71736242779302 \dots \times 10^{21} \text{ V/m}^2$	prediction
$A_{\text{ef}\nabla} = 9.7173624424(30) \times 10^{21} \text{ V/m}^2$	CODATA 2022, $\sigma = -4.87$
$A_{\text{ef}\nabla} = 9.7173624292(29) \times 10^{21} \text{ V/m}^2$	CODATA 2018, $\sigma = -0.49$
	$\Delta_{\text{precision}} = -0.02$
	$\Delta_{\text{scaled}} = +4.55$

$$A_{\text{ef}\nabla} = \varkappa_1^7 \left( \frac{m_e^3}{t_p^2 q_p m_p^2} \right) \left( 1 - \frac{1}{\sqrt{C_{CFP}}} \left( \frac{4\pi}{18} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{ef}\nabla} = \varkappa_1^7 \left( \frac{m_e^3}{t_p^2 q_p m_p^2} \right) \left( 1 - \mathcal{S} \left( \frac{2\pi}{32} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{ef}\nabla} = \varkappa_1^7 \left( \frac{m_e^3}{t_p^2 q_p m_p^2} \right) \left( 1 + \frac{s^2}{35} \left( \frac{32}{4\pi} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

**atomic unit of current**

-12.016420(11)

$$A_{\text{current}} = \varkappa_1^5 \left( \frac{q_p m_e}{t_p m_p} \right) \left( 1 - C_{\text{CFP}} \left( \frac{V_{\text{fe}}}{8} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $q_p$  = the Planck charge,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $m_p$  = the Planck mass,  $C_{\text{CFP}}$  = the fixed point of the hyperbolic cotangent,  $V_{\text{fe}}$  = the figure eight knot hyperbolic volume,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{current}} = 6.62361823748092 \dots \times 10^{-3} \text{ A}$	prediction
$A_{\text{current}} = 6.6236182375082(72) \times 10^{-3} \text{ A}$	CODATA 2022, $\sigma = -3.79$
$A_{\text{current}} = 6.623618237510(13) \times 10^{-3} \text{ A}$	CODATA 2018, $\sigma = -2.24$
	$\Delta_{\text{precision}} = +0.26$
	$\Delta_{\text{scaled}} = -0.15$

**atomic unit of magnetic flux density**

-12.7674(31)

$$A_{\text{mfd}} = \varkappa_1^3 \left( \frac{m_e^2}{t_p q_p m_p} \right) \left( 1 + \sqrt{8} \left( \frac{4\pi}{35} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $m_e$  = the electron mass,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $\pi$  = Archimedes' constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{mfd}} = 2.35051756770603 \dots \times 10^5 \text{ T}$	prediction
$A_{\text{mfd}} = 2.35051757077(73) \times 10^5 \text{ T}$	CODATA 2022, $\sigma = -4.20$
$A_{\text{mfd}} = 2.35051756758(71) \times 10^5 \text{ T}$	CODATA 2018, $\sigma = +0.18$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = +4.49$

$$A_{\text{mfd}} = \varkappa_1^3 \left( \frac{m_e^2}{t_p q_p m_p} \right) \left( 1 + G_{\text{Gi}} (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

## atomic unit of electric dipole moment

9.1408(16)

$$A_{\text{edm}} = \frac{1}{\varkappa_1} \left( \frac{l_p q_p m_p}{m_e} \right) \left( 1 + \frac{P}{L_2} \left( \frac{4\pi}{5!} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $P$  = the plastic constant,  $L_2$  = the 2<sup>nd</sup> lemniscate constant,  $\pi$  = Archimedes' constant,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{edm}} = 8.47835362016674 \dots \times 10^{-30} \text{ m} \cdot \text{C}$	prediction
$A_{\text{edm}} = 8.4783536198(13) \times 10^{-30} \text{ m} \cdot \text{C}$	CODATA 2022, $\sigma = +0.28$
$A_{\text{edm}} = 8.4783536255(13) \times 10^{-30} \text{ m} \cdot \text{C}$	CODATA 2018, $\sigma = -4.10$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -4.38$

$$A_{\text{edm}} = \frac{1}{\varkappa_1} \left( \frac{l_p q_p m_p}{m_e} \right) \left( 1 + \frac{L_2}{D_{Do}} \left( \frac{4\pi}{!5} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{edm}} = \frac{1}{\varkappa_1} \left( \frac{l_p q_p m_p}{m_e} \right) \left( 1 - \frac{18}{4} \left( \frac{V_{\text{fe}}}{4\pi} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{edm}} = \frac{1}{\varkappa_1} \left( \frac{l_p q_p m_p}{m_e} \right) \left( 1 - \frac{8}{7} \left( \frac{8}{4\pi} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{edm}} = \frac{1}{\varkappa_1} \left( \frac{l_p q_p m_p}{m_e} \right) \left( 1 - \frac{1}{7} \left( \frac{32}{2\pi} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{edm}} = \frac{1}{\varkappa_1} \left( \frac{l_p q_p m_p}{m_e} \right) \left( 1 - \frac{V_{\text{fe}}}{4} \left( \frac{18}{4\pi} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{edm}} = \frac{1}{\varkappa_1} \left( \frac{l_p q_p m_p}{m_e} \right) \left( 1 + 4 \left( \frac{V_{\text{fe}}}{35} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{mag}} = \frac{1}{\varkappa_1^2} \left( \frac{l_p^2 q_p^2 m_p^2}{m_e^3} \right) \left( 1 + L_{LL} \left( \frac{4\pi}{18} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $L_{LL}$  = the Laplace limit,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{mag}} = 7.89103656936017 \dots \times 10^{-29} \text{ J/T}^2$	prediction
$A_{\text{mag}} = 7.8910365794(49) \times 10^{-29} \text{ J/T}^2$	CODATA 2022, $\sigma = -2.05$
$A_{\text{mag}} = 7.8910366008(48) \times 10^{-29} \text{ J/T}^2$	CODATA 2018, $\sigma = -6.55$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = -4.45$

$$A_{\text{mag}} = \frac{1}{\varkappa_1^2} \left( \frac{l_p^2 q_p^2 m_p^2}{m_e^3} \right) \left( 1 + P \left( \frac{2\pi}{18} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{mag}} = \frac{1}{\varkappa_1^2} \left( \frac{l_p^2 q_p^2 m_p^2}{m_e^3} \right) \left( 1 - \frac{4}{7} \left( \frac{32}{4\pi} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{mag}} = \frac{1}{\varkappa_1^2} \left( \frac{l_p^2 q_p^2 m_p^2}{m_e^3} \right) \left( 1 + \frac{\omega_2}{2} \left( \frac{2\pi}{18} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{mag}} = \frac{1}{\varkappa_1^2} \left( \frac{l_p^2 q_p^2 m_p^2}{m_e^3} \right) \left( 1 + \frac{S}{2} \left( \frac{2\pi}{!5} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{mag}} = \frac{1}{\varkappa_1^2} \left( \frac{l_p^2 q_p^2 m_p^2}{m_e^3} \right) \left( 1 + \frac{1}{\sqrt{35}} \left( \frac{2\pi}{x_\infty} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{mag}} = \frac{1}{\varkappa_1^2} \left( \frac{l_p^2 q_p^2 m_p^2}{m_e^3} \right) \left( 1 - G_{\text{Gi}} \left( \frac{18}{4\pi} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

$$A_{\text{mag}} = \frac{1}{\varkappa_1^2} \left( \frac{l_p^2 q_p^2 m_p^2}{m_e^3} \right) \left( 1 - \sqrt{32 e^{1/e}} (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

## atomic unit of electric quadrupole moment

17.9074(31)

$$A_{\text{eqm}} = \frac{1}{\kappa_1^3} \left( \frac{l_p^2 q_p m_p^2}{m_e^2} \right) \left( 1 + \frac{2}{\sqrt{3}} \left( \frac{4\pi}{32} \right) (\kappa_2^2 - \kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $l_p$  = the Planck length,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $m_e$  = the electron mass,  $\pi$  = Archimedes' constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{\text{eqm}} = 4.48655151342649 \dots \times 10^{-40} \text{ m} \cdot \text{C}$	prediction
$A_{\text{eqm}} = 4.4865515185(14) \times 10^{-40} \text{ m} \cdot \text{C}$	CODATA 2022, $\sigma = -3.62$
$A_{\text{eqm}} = 4.4865515246(14) \times 10^{-40} \text{ m} \cdot \text{C}$	CODATA 2018, $\sigma = -9.87$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -4.36$

$$A_{\text{eqm}} = \frac{1}{\kappa_1^3} \left( \frac{l_p^2 q_p m_p^2}{m_e^2} \right) \left( 1 + \frac{7}{18} \left( \frac{8K}{2\pi} \right) (\kappa_2^2 - \kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

$$A_{\text{eqm}} = \frac{1}{\kappa_1^3} \left( \frac{l_p^2 q_p m_p^2}{m_e^2} \right) \left( 1 + \frac{\gamma}{2} \left( \frac{4\pi}{8} \right) (\kappa_2^2 - \kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

$$A_{\text{eqm}} = \frac{1}{\kappa_1^3} \left( \frac{l_p^2 q_p m_p^2}{m_e^2} \right) \left( 1 - 7 \left( \frac{4K}{18} \right) (\kappa_2^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

$$A_{\text{eqm}} = \frac{1}{\kappa_1^3} \left( \frac{l_p^2 q_p m_p^2}{m_e^2} \right) \left( 1 - \frac{12}{j_{0,1}} \left( \frac{4\pi}{!5} \right) (\kappa_2^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

$$A_{\text{eqm}} = \frac{1}{\kappa_1^3} \left( \frac{l_p^2 q_p m_p^2}{m_e^2} \right) \left( 1 - \frac{2\pi}{\sqrt{3}} \left( \frac{4\pi}{32} \right) (\kappa_2^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

## atomic unit of 1<sup>st</sup> hyperpolarizability

41.9099(46)

$$A_{1^{\text{st}} \text{ hp}} = \frac{1}{\kappa_1^{11}} \left( \frac{t_p^4 q_p^3 m_p^3}{l_p m_e^5} \right) \left( 1 - \frac{3}{\sqrt{2}} \left( \frac{4\pi}{8} \right) (\kappa_2^2 + \kappa_3^2 + \kappa_4^2) \boxtimes \right)$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $\pi$  = Archimedes' constant,  $\kappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\kappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\kappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{1^{\text{st}} \text{ hp}} = 3.20636129566261 \dots \times 10^{-53} \text{ m}^3 \cdot \text{C}^3 / \text{J}^2$	prediction
$A_{1^{\text{st}} \text{ hp}} = 3.2063612996(15) \times 10^{-53} \text{ m}^3 \cdot \text{C}^3 / \text{J}^2$	CODATA 2022, $\sigma = -2.62$
$A_{1^{\text{st}} \text{ hp}} = 3.2063613061(15) \times 10^{-53} \text{ m}^3 \cdot \text{C}^3 / \text{J}^2$	CODATA 2018, $\sigma = -6.96$
	$\Delta_{\text{precision}} = +0.00$
	$\Delta_{\text{scaled}} = -4.33$

$$A_{1^{\text{st}} \text{ hp}} = \frac{1}{\kappa_1^{11}} \left( \frac{t_p^4 q_p^3 m_p^3}{l_p m_e^5} \right) \left( 1 + \pi \left( \frac{V_{fe}}{6} \right) (\kappa_2^2 - \kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

$$A_{1^{\text{st}} \text{ hp}} = \frac{1}{\kappa_1^{11}} \left( \frac{t_p^4 q_p^3 m_p^3}{l_p m_e^5} \right) \left( 1 + \frac{D_d}{L_{LL}} \left( \frac{4\pi}{32} \right) (\kappa_2^2 - \kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

$$A_{1^{\text{st}} \text{ hp}} = \frac{1}{\kappa_1^{11}} \left( \frac{t_p^4 q_p^3 m_p^3}{l_p m_e^5} \right) \left( 1 + \frac{2}{D_{Do}} \left( \frac{4\pi}{32} \right) (\kappa_2^2 - \kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

$$A_{1^{\text{st}} \text{ hp}} = \frac{1}{\kappa_1^{11}} \left( \frac{t_p^4 q_p^3 m_p^3}{l_p m_e^5} \right) \left( 1 - \sqrt{\frac{7}{5}} \left( \frac{4\pi}{14} \right) (\kappa_2^2 - \kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

$$A_{1^{\text{st}} \text{ hp}} = \frac{1}{\kappa_1^{11}} \left( \frac{t_p^4 q_p^3 m_p^3}{l_p m_e^5} \right) \left( 1 + \frac{2\pi}{\sqrt{35}} (\kappa_2^2 - \kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

$$A_{1^{\text{st}} \text{ hp}} = \frac{1}{\kappa_1^{11}} \left( \frac{t_p^4 q_p^3 m_p^3}{l_p m_e^5} \right) \left( 1 - D_{Do} \left( \frac{18}{4\pi} \right) (\kappa_2^2 - \kappa_3^2 - \kappa_4^2) \boxtimes \right)$$

$$A_{2^{\text{nd}} \text{ hp}} = \frac{1}{\varkappa_1^{16}} \left( \frac{t_p^6 q_p^4 m_p^4}{l_p^2 m_e^7} \right) \left( 1 - \frac{12}{D_{Do}} \left( \frac{4\pi}{!5} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant,  $t_p$  = the Planck time,  $q_p$  = the Planck charge,  $m_p$  = the Planck mass,  $l_p$  = the Planck length,  $m_e$  = the electron mass,  $D_{Do}$  = the Dottie number,  $\pi$  = Archimedes' constant,  $!n$  = the derangement function,  $\varkappa_2$  = the 2<sup>nd</sup> hyperbolic partition constant,  $\varkappa_3$  = the 3<sup>rd</sup> hyperbolic partition constant,  $\varkappa_4$  = the 4<sup>th</sup> hyperbolic partition constant, and  $\boxtimes$  = the hyperbolic inversion boundary.

$A_{2^{\text{nd}} \text{ hp}} = 6.23537998029131 \dots \times 10^{-65} \text{ m}^4 \cdot \text{C}^4 / \text{J}^3$	prediction
$A_{2^{\text{nd}} \text{ hp}} = 6.2353799735(39) \times 10^{-65} \text{ m}^4 \cdot \text{C}^4 / \text{J}^3$	CODATA 2022, $\sigma = +1.74$
$A_{2^{\text{nd}} \text{ hp}} = 6.2353799905(38) \times 10^{-65} \text{ m}^4 \cdot \text{C}^4 / \text{J}^3$	CODATA 2018, $\sigma = -2.69$
	$\Delta_{\text{precision}} = -0.01$
	$\Delta_{\text{scaled}} = -4.47$

$$A_{2^{\text{nd}} \text{ hp}} = \frac{1}{\varkappa_1^{16}} \left( \frac{t_p^6 q_p^4 m_p^4}{l_p^2 m_e^7} \right) \left( 1 - P \left( \frac{!5}{4\pi} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

$$A_{2^{\text{nd}} \text{ hp}} = \frac{1}{\varkappa_1^{16}} \left( \frac{t_p^6 q_p^4 m_p^4}{l_p^2 m_e^7} \right) \left( 1 - \varphi \left( \frac{18}{2\pi} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

$$A_{2^{\text{nd}} \text{ hp}} = \frac{1}{\varkappa_1^{16}} \left( \frac{t_p^6 q_p^4 m_p^4}{l_p^2 m_e^7} \right) \left( 1 + P \left( \frac{14}{4\pi} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

$$A_{2^{\text{nd}} \text{ hp}} = \frac{1}{\varkappa_1^{16}} \left( \frac{t_p^6 q_p^4 m_p^4}{l_p^2 m_e^7} \right) \left( 1 + L_{LL} \left( \frac{14}{2\pi} \right) (\varkappa_2^2 - \varkappa_3^2 - \varkappa_4^2) \boxtimes \right)$$

$$A_{2^{\text{nd}} \text{ hp}} = \frac{1}{\varkappa_1^{16}} \left( \frac{t_p^6 q_p^4 m_p^4}{l_p^2 m_e^7} \right) \left( 1 - \frac{16}{j_{0,1}} \left( \frac{4\pi}{18} \right) (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

$$A_{2^{\text{nd}} \text{ hp}} = \frac{1}{\varkappa_1^{16}} \left( \frac{t_p^6 q_p^4 m_p^4}{l_p^2 m_e^7} \right) \left( 1 - \frac{18}{8} \left( \frac{2\pi}{s} \right)^4 (\varkappa_2^2 + \varkappa_3^2 + \varkappa_4^2) \boxtimes \right)$$

## proton rms charge radius

8.7541(46)

$$\frac{r_n}{r_+} = K = \sum_{n=1}^{\infty} \frac{\sin\left(n \frac{\pi}{2}\right)}{n^2}$$

Where  $K$  = Catalan's constant,  $\sin(x)$  = the sine function, and  $\pi$  = Archimedes' constant.

$$r_+ = 8.43431614404474 \dots \times 10^{-16} \text{ m}$$

$$r_+ = 8.4075(64) \times 10^{-16} \text{ m}$$

$$r_+ = 8.414(19) \times 10^{-16} \text{ m}$$

prediction

CODATA 2022,  $\sigma = +4.18$

CODATA 2018,  $\sigma = +1.05$

$$\Delta_{\text{precision}} = 0.47$$

$$\Delta_{\text{scaled}} = 0.36$$

## neutron radius

$$\frac{r_n}{r_e} = \left(\frac{4\pi}{\Gamma(5)}\right)^2 = \sum_{n=1}^{\infty} \frac{\cos\left(n \frac{\pi}{3}\right)}{n^2}$$

Where  $r_e$  = the classical electron radius,  $\pi$  = Archimedes' constant,  $\Gamma(x)$  = the gamma function, and  $\cos(x)$  = the cosine function.

$$r_n = 7.72554339835845 \dots \times 10^{-16} \text{ m}$$

$$r_n = 8 \times 10^{-16} \text{ m}$$

prediction

Povh, B.: Rith, K.: Scholz, C.: Zetsche, F. (2002)

## deuteron rms charge radius

$$\frac{r_e}{r_{de}} = \sqrt{\frac{2\pi}{\log(36)}}$$

Where  $r_e$  = the classical electron radius,  $r_{de}$  = the deuteron rms charge radius,  $\pi$  = Archimedes' constant, and  $\log(x)$  = the natural logarithm.

$r_{de} = 2.12812292371170 \dots \times 10^{-15} \text{ m}$	prediction
$r_{de} = 2.12778(27) \times 10^{-15} \text{ m}$	CODATA 2022, $\sigma = +1.26$
$r_{de} = 2.12799(74) \times 10^{-15} \text{ m}$	CODATA 2018, $\sigma = +0.18$
	$\Delta_{\text{precision}} = 0.44$
	$\Delta_{\text{scaled}} = 0.28$

## alpha particle rms charge radius

$$\frac{r_n}{r_\alpha} = \sqrt{\frac{C_U}{4}} \quad C_U = \sqrt{2} \left( \frac{2\pi}{2s} \right)$$

Where  $r_n$  = the neutron radius,  $r_\alpha$  = the alpha particle rms charge radius, and  $C_U$  = the ubiquitous constant,  $\pi$  = Archimedes' constant, and  $s$  = the arc length of the unit lemniscate.

$r_\alpha = 1.67865985670315 \dots \times 10^{-15} \text{ m}$	prediction
$r_\alpha = 1.6785(21) \times 10^{-15} \text{ m}$	CODATA 2022, $\sigma = +0.05$
$r_\alpha = 1.6785(21) \times 10^{-15} \text{ m}$	CODATA 2018, $\sigma = +0.05$
	$\Delta_{\text{precision}} = 0.00$
	$\Delta_{\text{scaled}} = 0.00$

## fine-structure constant

$$\alpha = \kappa_1^2$$

Where  $\kappa_1$  = the 1<sup>st</sup> hyperbolic partition constant.

$$\alpha = 7.29735257295520 \dots \times 10^{-3} \quad \text{prediction}$$
$$\alpha = 7.2973525684(14) \times 10^{-3} \quad \text{average 11-digit measure, } \sigma = +3.21$$

$\alpha = 7.29735256491(80) \times 10^{-3}$	Fan et al. 2023, $\sigma = +10.05$
$\alpha = 7.2973525643(11) \times 10^{-3}$	CODATA 2022, $\sigma = +7.82$
$\alpha = 7.2973525628(06) \times 10^{-3}$	Morel et al. 2020, $\sigma = +16.83$
$\alpha = 7.2973525693(11) \times 10^{-3}$	CODATA 2018, $\sigma = +3.27$
$\alpha = 7.2973525713(14) \times 10^{-3}$	Parker, Yu, et al. 2018, $\sigma = +1.14$
$\alpha = 7.2973525657(18) \times 10^{-3}$	Aoyama et al. 2017, $\sigma = +4.00$
$\alpha = 7.2973525664(17) \times 10^{-3}$	CODATA 2014, $\sigma = +3.82$
$\alpha = 7.2973525717(48) \times 10^{-3}$	Bouchendira 2010, $\sigma = +0.25$
$\alpha = 7.2973525698(24) \times 10^{-3}$	CODATA 2010, $\sigma = +1.29$
$\alpha = 7.2973525692(27) \times 10^{-3}$	Gabrielse, Hanneke 2008, $\sigma = +1.37$
$\alpha = 7.2973525700(52) \times 10^{-3}$	Gabrielse 2007, $\sigma = +0.56$
$\alpha = 7.2973525376(50) \times 10^{-3}$	CODATA 2006, $\sigma = +7.06$
$\alpha = 7.297352568(24) \times 10^{-3}$	CODATA 2002, $\sigma = +2.04$
$\alpha = 7.297352533(27) \times 10^{-3}$	CODATA 1998, $\sigma = +1.44$
$\alpha = 7.297352582(27) \times 10^{-3}$	Kinoshita 1998, $\sigma = -0.37$
$\alpha = 7.29735308(33) \times 10^{-3}$	CODATA 1986, $\sigma = -1.55$
$\alpha = 7.2973461(81) \times 10^{-3}$	CODATA 1973, $\sigma = +0.79$
$\alpha = 7.297351(11) \times 10^{-3}$	CODATA 1969, $\sigma = +0.09$

\* Note Fan et al. 2023 is not an experimentally measured value. It is a value determined by *current theory* based on an experimentally determined value of the *electron magnetic moment*.

\*\* CODATA values are computed by averaging other measurements; they are not independent experiments.

## electron charge

$$e = \varkappa_1 q_p$$

Where  $\varkappa_1$  = the 1<sup>st</sup> hyperbolic partition constant, and  $q_p$  = the Planck charge.

$e = 1.60217657405973 \dots \times 10^{-19} \text{ C}$	prediction
$e = 1.602176634 \times 10^{-19} \text{ C}$	CODATA 2022, 8-digit match
$e = 1.602176634 \times 10^{-19} \text{ C}$	CODATA 2018, 8-digit match
$e = 1.602176565(35) \times 10^{-19} \text{ C}$	CODATA 2010, $\sigma = +0.26$
	$\Delta_{\text{precision}} = 0.00$
	$\Delta_{\text{scaled}} = 0.00$

Also known as the *elementary charge*, and the *atomic unit of charge*.

## W to Z mass ratio

$$\frac{m_W}{m_Z} = \log(1 + \sqrt{2})$$

Where  $m_W$  = the W boson mass,  $m_Z$  = the Z boson mass, and  $\log(x)$  = the natural logarithm.

$m_W/m_Z = 0.881373587019543 \dots$	prediction
$m_W/m_Z = 0.88145(13)$	CODATA 2022, $\sigma = -0.62$
$m_W/m_Z = 0.88153(17)$	CODATA 2018, $\sigma = -0.94$
	$\Delta_{\text{precision}} = 0.12$
	$\Delta_{\text{scaled}} = 0.47$

$$\log(1 + \sqrt{2}) = P_{up} - \sqrt{2}$$

$$\frac{m_W}{m_Z} + \sqrt{2} = P_{up}$$

## weak mixing angle

$$\frac{m_W}{m_Z} = \cos(\theta_W) \qquad 1 - \left(\frac{m_W}{m_Z}\right)^2 = \sin^2(\theta_W)$$

Where  $\cos(x)$  = the cosine function,  $\sin(x)$  = the sine function,  $m_W$  = the mass of the W boson, and  $m_Z$  = the mass of the Z boson.

$\sin^2(\theta_W) = 0.223180600010430 \dots$	prediction
$\sin^2(\theta_W) = 0.22305(23)$	CODATA 2022, $\sigma = +0.57$
$\sin^2(\theta_W) = 0.22290(30)$	CODATA 2018, $\sigma = +0.93$
	$\Delta_{\text{precision}} = 0.12$
	$\Delta_{\text{scaled}} = 0.50$